

# Data Tables for Lorentz and CPT Violation

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This work tabulates measured and derived values of coefficients for Lorentz and CPT violation in the Standard-Model Extension. Summary tables are extracted listing maximal attained sensitivities in the matter, photon, and gravity sectors. Tables presenting definitions and properties are also compiled.

## I. INTRODUCTION

Recent years have seen a renewed interest in experimental tests of Lorentz and CPT symmetry. Observable signals of Lorentz and CPT violation can be described in a model-independent way using effective field theory [1].

The general realistic effective field theory for Lorentz violation is called the Standard-Model Extension (SME) [2, 3]. It includes the Standard Model coupled to General Relativity along with all possible operators for Lorentz violation. Both global [2] and local [3] Lorentz violation are incorporated. Since CPT violation in realistic field theories is accompanied by Lorentz violation [4], the SME also describes general CPT violation. Reviews of the SME can be found in Refs. [5, 6].

Each Lorentz-violating term in the Lagrange density of the SME is constructed as the coordinate-independent product of a coefficient for Lorentz violation with a Lorentz-violating operator. The Lorentz-violating physics associated with any operator is therefore controlled by the corresponding coefficient, and so any experimental signal for Lorentz violation can be expressed in terms of one or more of these coefficients.

The Lorentz-violating operators in the SME are systematically classified according to their mass dimension, and operators of arbitrarily large dimension can appear. At any fixed dimension, the operators are finite in number and can in principle be enumerated. A limiting case of particular interest is the minimal SME, which can be viewed as the restriction of the SME to include only Lorentz-violating operators of mass dimension four or less. The corresponding coefficients for Lorentz violation are dimensionless or have positive mass dimension.

The results summarized here concern primarily but not exclusively the coefficients for Lorentz violation in the minimal SME. We compile data tables for these SME coefficients, including both existing experimental measurements and some theory-derived limits, and we provide tables listing some relevant definitions and properties. We also extract summary tables listing our best estimates for the maximal attained sensitivities in three sectors: ordi-

nary matter (electrons, protons, and neutrons), photons, and gravity. The tables include results available from the literature up to December 31, 2009. More recent papers can be found online [7].

The order of the tables is as follows. Table I contains a list of all tables. The three summary tables are presented next, Tables II, III, and IV. These are followed by the data tables, Tables V to XV. The properties tables appear last, Tables XVI to XXIV.

A description of the summary tables is given in Sec. II. Information about the format and content of the data tables is presented in Sec. III, while Sec. IV provides an overview of the properties tables. The bibliography for the text and all the tables follows Sec. IV.

## II. SUMMARY TABLES

Three summary tables are provided (Tables II, III, IV), listing maximal experimental sensitivities attained for coefficients in the matter, photon, and gravity sectors of the minimal SME. To date, there is no compelling experimental evidence supporting Lorentz violation. A few measurements suggest nonzero coefficients at weak confidence levels. These latter results are excluded from the summary tables but are listed in the data tables.

In these three summary tables, each displayed sensitivity value represents our conservative estimate of a  $2\sigma$  limit, given to the nearest order of magnitude, on the modulus of the corresponding coefficient. In a few cases, tighter results may exist when suitable theoretical assumptions are adopted; these results can be found in the data tables that follow. Where observations involve a linear combination of coefficients in the tables, the displayed sensitivity for each coefficient assumes for definiteness that no other coefficient contributes. Some caution is therefore advisable in applying the results in these summary tables to situations involving two or more nonzero coefficient values. Care in applications is also required because under some circumstances certain coefficients can be intrinsically unobservable or can be absorbed into others by field or coordinate redefinitions.

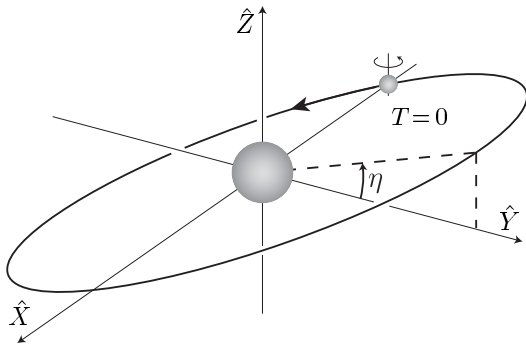


FIG. 1: Standard Sun-centered inertial reference frame [8].

In presenting the physical sensitivities, we adopt natural units with  $\hbar = c = \epsilon_0 = 1$  and express mass units in GeV. Our values are reported in the standard Sun-centered inertial reference frame widely used in the literature. This frame is illustrated in Fig. 1. It has  $Z$  axis directed north and parallel to the rotational axis of the Earth. The  $X$  axis originates from the Sun and points towards the vernal equinox, while the  $Y$  axis completes a right-handed system. The time coordinate  $T$  is fixed with origin at the 2000 vernal equinox. Details about this frame, including transformations to other frames, can be found in Section IIIA of Ref. [9].

Table II lists the maximal attained sensitivities involving protons, neutrons, and electrons. For each distinct massive spin-half Dirac fermion in the minimal SME in Minkowski spacetime, there are 44 independent observable combinations of coefficients for Lorentz violation in the nonrelativistic limit. Of these, 20 also control CPT violation. The 44 combinations are conventionally chosen as the tilde coefficients shown. The definitions of these 44 tilde coefficients in terms of minimal SME coefficients are listed in Table XVIII. All the definitions appear elsewhere in the literature [8] except the four combinations  $\tilde{b}_j^*$  and  $\tilde{c}_{TT}$ . In Table II, all tilde coefficients have dimensions of GeV in natural units. A dash indicates that no sensitivity to the coefficient has been identified to date.

Table III displays the maximal attained sensitivities to coefficients for Lorentz violation in the photon sector of the minimal SME. There are 23 observable coefficient combinations for photons, of which four also control CPT violation. The 19 tilde coefficients listed in the table are conventional combinations of the 19 dimensionless coefficients in the minimal SME. The remaining entries in the table concern combinations of the four coefficients controlling CPT violation, which have dimensions of GeV in natural units. The definitions of all 23 combinations are taken from the literature [9, 10] and are provided in Table XIX.

Table IV displays the maximal attained sensitivities to certain coefficients for Lorentz violation involving the

gravity sector of the minimal SME. Two classes of coefficients can be distinguished in this context: ones appearing in the matter sector, and ones appearing in the pure-gravity sector. For the first class, Table IV lists the 12 observables involving the coefficients  $\tilde{a}_\mu^e$ ,  $\tilde{a}_\mu^p$ , and  $\tilde{a}_\mu^n$  for electrons, protons, and neutrons, respectively. These observables are associated with CPT-odd operators and have dimensions of GeV in natural units. The prefactor  $\alpha$  is a model-dependent number [11]. For the second class, the table displays nine combinations of the nine dimensionless coefficients for Lorentz violation  $\tilde{\mathfrak{S}}^{\mu\nu}$ . Additional sets of coefficients involving the gravity sector exist, but no sensitivities to them have been identified to date.

### III. DATA TABLES

We present 11 data tables compiled from the existing literature. Of these, 10 tables include results for various sectors of the minimal SME: the electron sector (Table V), the proton sector (Table VI), the neutron sector (Table VII), the photon sector (Table VIII), the charged-lepton sector (Table IX), the neutrino sector (Table X), the meson sector (Table XI), the electroweak sector (Table XII), the gluon sector (Table XIII), and the gravity sector (Table XIV). The remaining table (Table XV) lists existing bounds on nonminimal coefficients for Lorentz violation in the photon sector.

Each of these 11 data tables contains four columns. The first column lists the coefficients for Lorentz violation or their relevant combinations. Results for coefficients of the same generic type are grouped together. Certain results involve combinations of coefficients across more than one sector; each of these has been listed only once in the table deemed most appropriate. Some minor changes in notation or format have been introduced as needed, but for the most part the results are quoted as they appear in the cited references. Definitions for standard combinations of coefficients are provided in the properties tables that follow. A few authors use unconventional notation; where immediate, the match to the standard notation is shown.

The second column contains the measurements and bounds, presented in the same form as documented in the literature. For each generic type of coefficient, the results are listed in reverse chronological order. If no significant figures appear in the quoted limit on an absolute value, the order of magnitude of the limit is given as a power of 10.

The third column contains a succinct reminder of the physical context in which the bound is extracted, while the fourth column contains the source citations. The reader is referred to the latter for details of experimental and theoretical procedures, assumptions underlying the results, definitions of unconventional notations, and other relevant information. Results deduced on theoret-

ical grounds are distinguished from those obtained via direct experimental measurement by an asterisk placed after the citation.

Tables V, VI, and VII contain data for the electron, proton, and neutron sectors, respectively. Each table is divided into sections focusing sequentially on combinations involving the coefficients  $b_\nu$ ,  $c_{\mu\nu}$ ,  $H_{\mu\nu}$ ,  $d_{\mu\nu}$ , and  $g_{\mu\nu\lambda}$ . Standard definitions for these coefficients and their combinations are provided in Tables XVI and XVIII. Some results depend on  $\eta \simeq 23.5^\circ$ , which is the angle between the equatorial and ecliptic planes in the solar system. Note that the existing bound on the combination of observables involving  $a_\nu^e$ ,  $a_\nu^p$ , and  $a_\nu^n$  is obtained from gravitational experiments and is listed with the gravity-sector results in Table XIV.

Table VIII presents the photon-sector data. Most of the combinations of coefficients for Lorentz violation appearing in the first column are defined in Tables XIX and XVI. The alternative combinations  $k_{(V)jm}^{(3)}$ ,  $k_{(E)jm}^{(4)}$ , and  $k_{(B)jm}^{(4)}$  arise from analyses [10, 12, 13] using spin-weighted spherical harmonics. The factor of  $\beta_\oplus$  appearing in some places is the speed of the Earth in the standard Sun-centered reference frame, which is about  $10^{-4}$  in natural units.

Tables IX, X, and XI list measurements and bounds on coefficients for Lorentz violation involving second- and third-generation fermions in the minimal SME. Results for muons and tau leptons are in Table IX, while those for neutrinos are in Table X. For both these tables, many of the coefficients appearing in the first column are specified in the lepton sector of Table XX. The neutrino results in Table X are obtained in the context of various simplified models, as discussed in the references. Experimental sensitivities to coefficients for operators involving second- and third-generation quark fields are presently limited to mesons and are presented in Table XI. The coefficients appearing in this table are composite quantities defined in the corresponding references. They are effective coefficients for which complete analytical expressions are as yet unknown, formed from certain quark-sector coefficients appearing in Table XX and from other quantities arising from the quark binding in the mesons.

Tables XII and XIII concern coefficients in the gauge sectors of the minimal SME. Results for the electroweak sector are listed in Table XII, while those for the gluon sector are in Table XIII. The coefficients for the electroweak sector are defined in the gauge and Higgs sections of Table XXI. The gluon-sector coefficient is the analogue of the corresponding photon-sector coefficient defined in Table XIX. To date, all results for the gauge sector are deduced from theoretical considerations.

Table XIV presents measurements and bounds concerning the gravity sector of the minimal SME. The specific combinations of coefficients in the pure-gravity sector that appear in the first column are defined in the

references. They are expressed in terms of the coefficients for Lorentz violation listed in the gravity section of Table XXI.

The final data table, Table XV, contains a compilation of some measurements and bounds on coefficients for Lorentz violation in the nonminimal SME. Attention is restricted to the photon sector, in which results are available for a variety of nonrenormalizable operators of dimensions 5, 6, 7, 8, and 9. A convenient basis for classifying operators of dimension  $d$  is given by the spin-weighted spherical harmonics [13]. The corresponding coefficients are listed in Table XXIV. Some constraints have been obtained for the vacuum coefficients for Lorentz violation, which are  $c_{(I)jm}^{(d)}$ ,  $k_{(E)jm}^{(d)}$ ,  $k_{(B)jm}^{(d)}$  for even  $d$  and  $k_{(V)jm}^{(d)}$  for odd  $d$ , where the subscripts  $jm$  label the angular-momentum quantum numbers. In the first column of Table XV, the usual spherical harmonics  ${}_0Y_{jm}$  are evaluated at specified angles, which are the celestial coordinates of certain astrophysical sources. None of the vacuum-orthogonal coefficients for Lorentz violation have been measured to date.

#### IV. PROPERTIES TABLES

Nine properties tables are provided, listing various features and definitions related to Lorentz violation. Four tables concern the terms in the restriction of the minimal SME to quantum electrodynamics (QED) in Riemann spacetime. For this theory, which is called the minimal QED extension, the tables include information about the operator structure (Table XVI), the action of discrete symmetries (Table XVII), and some useful coefficient combinations (Table XVIII and Table XIX). Two tables contain information about the matter sector (Table XX) and the gauge and gravity sectors (Table XXI) of the minimal SME in Riemann-Cartan spacetime. Another table (Table XXII) summarizes some features of the coefficients for Lorentz violation in the neutrino sector. The two remaining tables (Table XXIII and Table XXIV) provide information about the operator structure and the spherical coefficients for Lorentz violation in the nonminimal photon sector.

For these properties tables, our primary conventions are those of Ref. [3]. Greek indices  $\mu, \nu, \lambda, \dots$  refer to curved-spacetime coordinates and Latin indices  $a, b, c, \dots$  to local Lorentz coordinates. The vierbein formalism [14], which relates the two sets of coordinates, is adopted to facilitate the description of spinors on the spacetime manifold. The determinant  $e$  of the vierbein  $e_\mu^a$  is related to the determinant  $g$  of the metric  $g_{\mu\nu}$  by  $e = \sqrt{-g}$ . The conventions for the Dirac matrices  $\gamma^a$  are given in Appendix A of Ref. [3]. The Newton gravitational constant  $G_N$  enters as the combination  $\kappa \equiv 8\pi G_N$ , and it has dimensions of inverse mass squared.

In the Minkowski-spacetime limit, the metric  $g_{\mu\nu}$  is written  $\eta_{\mu\nu}$  with diagonal entries  $(-1, 1, 1, 1)$ . For decompositions into time and space components, we adopt the Sun-centered frame of Fig. 1 and use indices  $J, K, L, \dots$  to denote the three spatial components  $X, Y, Z$ . The sign of the antisymmetric tensor  $\epsilon_{\kappa\lambda\mu\nu}$  is fixed via the component  $\epsilon_{TXYZ} = +1$ , and the antisymmetric symbol in three spatial dimensions is defined with  $\epsilon_{XYZ} = +1$ . Note that some of the literature on the SME in Minkowski spacetime adopts a metric  $\eta_{\mu\nu}$  of opposite sign, following the common present usage in quantum physics instead of the one in relativity. Under this alternative convention, terms in the Lagrange density with an odd number of metric contractions have opposite signs to those appearing in this work.

Table XVI concerns the minimal QED extension, for which the basic nongravitational fields are a Dirac fermion  $\psi$  and the photon  $A_\mu$ . The electromagnetic field-strength tensor is  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ . The pure-gravity sector involves the Riemann tensor  $R_{\kappa\lambda\mu\nu}$ , the Ricci tensor  $R_{\mu\nu}$ , the curvature scalar  $R$ , and the cosmological constant  $\Lambda$ . The spacetime covariant derivative  $D_\mu$  corrects local Lorentz indices using the spin connection, corrects spacetime indices using the Cartan connection, and contains the usual gauge field  $A_\mu$  for the photon. The notation  $\overset{\leftrightarrow}{D}_\mu$  is an abbreviation for the difference of two terms, the first with derivative acting to the right and the second acting to the left. Note that Table XVI is restricted to the zero-torsion limit of the minimal SME. The general case [3] involves additional operators constructed with the torsion tensor  $T^\alpha_{\mu\nu}$ . The Minkowski-spacetime limit of QED with nonzero torsion contains terms that mimic Lorentz violation, so searches for Lorentz violation can be used to bound components of the torsion tensor [15].

In Table XVI, each line specifies one term in the Lagrange density for the QED extension in Riemann spacetime. Both conventional QED terms and ones with Lorentz violation are included. The first column indicates the sector to which the term belongs. The second column lists the coefficient controlling the corresponding operator. Note the standard use of an upper-case letter for the coefficient  $H_{\mu\nu}$ , which distinguishes it from the metric fluctuation  $h_{\mu\nu}$ . The third column shows the number of components for the coefficient. The next three columns list the operator, its mass dimension, and the vierbein factor contracting the coefficient and the operator. The final two columns list the properties of the term under CPT and Lorentz transformations. A CPT-even operator is indicated by a plus sign and a CPT-odd one by a minus sign, while terms violating Lorentz invariance are identified by a check.

Table XVII lists the properties under discrete-symmetry transformations of the Lorentz-violating operators in the minimal QED extension [16]. The seven

transformations considered are charge conjugation C, parity inversion P, time reversal T, and their combinations CP, CT, PT, and CPT. The first column specifies the operator by indicating its corresponding coefficient. Each of the other columns concerns one of the seven transformations. An even operator is indicated by a plus sign and an odd one by a minus sign. The table contains eight rows, one for each of the eight possible combinations of signs under C, P, and T.

Table XVIII lists the definitions of the 44 combinations of coefficients for Lorentz violation that frequently appear in experimental analyses involving the fermion sector of the minimal QED extension in Minkowski spacetime in the nonrelativistic limit. These combinations are conventionally denoted by tilde coefficients, listed in the first column of the table. Note that six of these combinations,  $\tilde{c}_X$ ,  $\tilde{c}_Y$ ,  $\tilde{c}_Z$ ,  $\tilde{g}_{TX}$ ,  $\tilde{g}_{TY}$ , and  $\tilde{g}_{TZ}$ , are denoted as  $\tilde{c}_{Q,Y}$ ,  $\tilde{c}_{Q,X}$ ,  $\tilde{c}_{XY}$ ,  $\tilde{g}_{Q,Y}$ ,  $\tilde{g}_{Q,X}$ , and  $\tilde{g}_{XY}$ , respectively, in some early publications. The definitions in the table are given for a generic fermion of mass  $m$ . Most applications in the literature involve electrons, protons, or neutrons, for which the corresponding mass is understood. The final column lists the number of independent components of each coefficient. For ordinary matter involving protons, electrons, and neutrons, there are therefore 132 independent observable coefficients for Lorentz violation in the minimal QED sector of the SME in Minkowski spacetime.

Table XIX presents definitions for certain combinations of the 23 coefficients for Lorentz violation in the photon sector of the minimal QED extension in Minkowski spacetime. This table has three sections. The first section consists of five rows listing 19 widely used combinations of the 19 coefficients for CPT-even Lorentz violation. The second section provides 10 alternative combinations involving the 10 CPT-even Lorentz-violating operators relevant to leading-order birefringence [9]. The third section lists four combinations of the four coefficients for CPT-odd Lorentz violation. These combinations appear when a basis of spin-weighted spherical harmonics is adopted.

Table XX concerns the fermion-sector terms in the Lagrange density of the minimal SME in Riemann-Cartan spacetime. The column headings are similar to those in Table XVI. In the lepton sector, the left- and right-handed leptons are denoted by  $L_A$  and  $R_A$ , where  $A$  is the generation index. The  $SU(2)$  doublet  $L_A$  includes the three neutrino fields  $\nu_e$ ,  $\nu_\mu$ ,  $\nu_\tau$  and the left-handed components of the three charged leptons  $e$ ,  $\mu$ , and  $\tau$ . The  $SU(2)$  singlet  $R_A$  contains the right-handed components of  $e$ ,  $\mu$ , and  $\tau$ . The derivative  $D_\mu$  is both spacetime and  $SU(3) \times SU(2) \times U(1)$  covariant. The quark fields are denoted  $U_A$ ,  $D_A$ , and  $Q_A$ , where  $A$  is the generation index. The right-handed components of the  $u$ ,  $c$ , and  $t$  quarks are the  $SU(2)$  singlets  $U_A$ , while the right-handed components of  $d$ ,  $s$ , and  $b$  are the  $SU(2)$  singlets  $D_A$ . The

six left-handed quark fields are contained in the  $SU(2)$  doublet  $Q_A$ . The Yukawa sector involves terms coupling the Higgs doublet  $\phi$  to the leptons and to the quarks. The conventional Yukawa-coupling matrices are denoted  $(G_L)_{AB}$ ,  $(G_U)_{AB}$ , and  $(G_D)_{AB}$ . The hermitian conjugate of an operator is abbreviated h.c. in the table.

Table XXI presents information about the Higgs, gauge, and pure-gravity sectors for the Lagrange density of the minimal SME in Riemann-Cartan spacetime. The structure of the table is the same as that of Table XX. As before,  $D_\mu$  is both a spacetime and an  $SU(3) \times SU(2) \times U(1)$  covariant derivative. The complex Higgs field is denoted  $\phi$ , the  $SU(3)$  color gauge fields and the  $SU(2)$  gauge fields are the hermitian adjoint matrices  $G_\mu$  and  $W_\mu$ , respectively, while the  $U(1)$  hypercharge gauge field is the singlet  $B_\mu$ . Each gauge field has an associated field strength, denoted  $G_{\mu\nu}$  for the strong interactions,  $W_{\mu\nu}$  for the weak interactions, and  $B_{\mu\nu}$  for the hypercharge. The pure-gravity sector of Table XXI differs from that in Table XVI only in the addition of terms involving the torsion field  $T^\alpha_{\mu\nu}$ .

The minimal SME in Riemann-Cartan spacetime described in Tables XX and XXI can be reduced to the minimal QED in Riemann spacetime described in Table XVI as follows. For the gauge sector, including the covariant derivatives, remove all the gauge fields except the charge  $U(1)$  field in the photon limit  $B_\mu \rightarrow A_\mu$ , and remove all the Higgs terms. For the gravity sector, remove all the torsion terms. For the fermion sector, restrict the lepton generation index to a single value, remove all quark and neutrino terms, and replace the Yukawa-coupling terms with the relevant fermion mass terms.

Table XXII concerns the neutrino sector of the SME, including both neutrino masses and Lorentz-violating terms. For the latter, we restrict attention to terms of mass dimension four or less that involve three generations of neutrinos and antineutrinos, allowing for possible violations of  $SU(3) \times SU(2) \times U(1)$  gauge symmetry and lepton number [17]. In the table, the first row involves the usual neutrino mass matrix  $\tilde{m}_{AB}$ , where the indices  $A, B$  take values  $e, \mu$ , and  $\tau$ , while the other rows concern coefficients for Lorentz violation. The first column lists the coefficients, and the second gives the dimension of the corresponding operators in the Lagrange density. The third column indicates generically the type of neutrino oscillations controlled by the coefficients. The final two columns list the properties of the operators under CPT and Lorentz transformations.

Table XXIII provides information about the nonminimal photon sector of the full SME in Minkowski spacetime. The relevant part of the Lagrange density includes operators of arbitrary dimension  $d$  that are both gauge invariant and quadratic in the photon field  $A_\mu$  [13]. The structure of the table is similar to those adopted for Tables XVI, XX, and XXI, with each row associated with a term in the Lagrange density. The first column lists

the coefficient for Lorentz violation, while the second column counts its independent components. The next three columns provide the corresponding operator appearing in the Lagrange density, its mass dimension, and the factor contracting the coefficient and the operator. The last two columns list the properties of the operator under CPT and Lorentz transformations, using the same conventions as Table XVI.

Three sections appear in Table XXIII, separated by horizontal lines. The first section concerns the conventional Lorentz-preserving Maxwell term in the Lagrange density for the photon sector. The second and third sections concern coefficients associated with operators of odd and even dimensions  $d$ , respectively. Each of these sections has three rows for the lowest three values of  $d$ , along with a final row applicable to the case of general  $d$ . The notation for the coefficients incorporates a superscript specifying the dimension  $d$  of the corresponding operator. Note that the mass dimension of the coefficients is  $4 - d$ . In each section, the first row describes terms in the minimal SME, and the match is provided between the general notation for nonminimal coefficients and the standard notation used for the minimal SME in Table XVI. In the case of mass dimension four, there are 19 independent Lorentz-violating operators. However, for this case the number in the second column is listed as  $19 + 1$  to allow for an additional Lorentz-preserving trace term, which maintains consistency with the expression for general  $d$  in the last row.

Table XXIV summarizes properties of spherical coefficients for Lorentz violation in the nonminimal photon sector of the full SME in Minkowski spacetime [13]. The spherical coefficients are combinations of the coefficients listed in Table XXIII that are of particular relevance for observation and experiment. They can be separated into two types. One set consists of vacuum coefficients that control leading-order effects on photon propagation in the vacuum, including birefringence and dispersion. The complementary set contains the vacuum-orthogonal coefficients, which leave photon propagation in the vacuum unaffected at leading order. The two parts of Table XXIV reflect this separation, with the part above the horizontal line involving the vacuum coefficients and the part below involving the vacuum-orthogonal ones.

In Table XXIV, the first column of the table identifies the type of spherical coefficients, while the second column lists the specific coefficient. The coefficient notation reflects properties of the corresponding operator. Coefficients associated with operators leaving unchanged the leading-order photon propagation in the vacuum are distinguished by a negation diacritic  $\neg$ . A symbol  $k$  denotes coefficients for birefringent operators, while  $c$  denotes nonbirefringent ones. The superscript  $d$  refers to the operator mass dimension, while the the the subscripts  $n, j, m$  determine the frequency or wavelength dependence, the total angular momentum, and the  $z$ -component of

the angular momentum, respectively. The superscripts  $E$  and  $B$  refer to the parity of the operator, while the numerals 0, 1, or 2 preceding  $E$  or  $B$  refer to the spin weight. Note that the photon-sector coefficients in the minimal SME correspond to the vacuum coefficients with  $d = 3, 4$ . The third, fourth, and fifth columns of Table XXIV provide the allowed ranges of the dimension  $d$  and of the indices  $n$  and  $j$ . The index  $m$  can take values ranging from  $-j$  to  $j$  in unit increments. The final column gives the number of independent coefficient components for each operator of dimension  $d$ .

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TABLE I: List of tables

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TABLE II: Maximal sensitivities for the matter sector

Coefficient	Proton	Neutron	Electron
$\tilde{b}_X$	$10^{-27}$ GeV	$10^{-31}$ GeV	$10^{-31}$ GeV
$\tilde{b}_Y$	$10^{-27}$ GeV	$10^{-31}$ GeV	$10^{-31}$ GeV
$\tilde{b}_Z$	—	—	$10^{-30}$ GeV
$\tilde{b}_T$	—	$10^{-27}$ GeV	$10^{-27}$ GeV
$\tilde{b}_J^* (J = X, Y, Z)$	—	—	—
$\tilde{c}_-$	$10^{-25}$ GeV	$10^{-27}$ GeV	$10^{-19}$ GeV
$\tilde{c}_Q$	$10^{-22}$ GeV	—	$10^{-19}$ GeV
$\tilde{c}_X$	$10^{-25}$ GeV	$10^{-25}$ GeV	$10^{-19}$ GeV
$\tilde{c}_Y$	$10^{-25}$ GeV	$10^{-25}$ GeV	$10^{-19}$ GeV
$\tilde{c}_Z$	$10^{-24}$ GeV	$10^{-27}$ GeV	$10^{-19}$ GeV
$\tilde{c}_{TX}$	$10^{-20}$ GeV	—	$10^{-18}$ GeV
$\tilde{c}_{TY}$	$10^{-20}$ GeV	—	$10^{-18}$ GeV
$\tilde{c}_{TZ}$	$10^{-21}$ GeV	—	$10^{-20}$ GeV
$\tilde{c}_{TT}$	—	—	$10^{-18}$ GeV
$\tilde{d}_+$	—	$10^{-27}$ GeV	$10^{-27}$ GeV
$\tilde{d}_-$	—	$10^{-27}$ GeV	$10^{-27}$ GeV
$\tilde{d}_Q$	—	$10^{-27}$ GeV	$10^{-27}$ GeV
$\tilde{d}_{XY}$	—	$10^{-27}$ GeV	$10^{-27}$ GeV
$\tilde{d}_{YZ}$	—	$10^{-26}$ GeV	$10^{-27}$ GeV
$\tilde{d}_{ZX}$	—	—	$10^{-26}$ GeV
$\tilde{d}_X$	$10^{-25}$ GeV	$10^{-29}$ GeV	$10^{-22}$ GeV
$\tilde{d}_Y$	$10^{-25}$ GeV	$10^{-28}$ GeV	$10^{-22}$ GeV
$\tilde{d}_Z$	—	—	$10^{-19}$ GeV
$\tilde{H}_{XT}$	—	$10^{-26}$ GeV	$10^{-27}$ GeV
$\tilde{H}_{YT}$	—	$10^{-27}$ GeV	$10^{-27}$ GeV
$\tilde{H}_{ZT}$	—	$10^{-27}$ GeV	$10^{-27}$ GeV
$\tilde{g}_T$	—	$10^{-27}$ GeV	$10^{-27}$ GeV
$\tilde{g}_c$	—	$10^{-27}$ GeV	$10^{-27}$ GeV
$\tilde{g}_Q$	—	—	—
$\tilde{g}_-$	—	—	—
$\tilde{g}_{TJ} (J = X, Y, Z)$	—	—	—
$\tilde{g}_{XY}$	—	—	—
$\tilde{g}_{YX}$	—	—	—
$\tilde{g}_{ZX}$	—	—	—
$\tilde{g}_{XZ}$	—	—	—
$\tilde{g}_{YZ}$	—	—	—
$\tilde{g}_{ZY}$	—	—	—
$\tilde{g}_{DX}$	$10^{-25}$ GeV	$10^{-29}$ GeV	$10^{-22}$ GeV
$\tilde{g}_{DY}$	$10^{-25}$ GeV	$10^{-28}$ GeV	$10^{-22}$ GeV
$\tilde{g}_{DZ}$	—	—	—

TABLE III: Maximal sensitivities for the photon sector

Coefficient	Sensitivity
$(\tilde{\kappa}_{e+})^{XY}$	$10^{-32}$
$(\tilde{\kappa}_{e+})^{XZ}$	$10^{-32}$
$(\tilde{\kappa}_{e+})^{YZ}$	$10^{-32}$
$(\tilde{\kappa}_{e+})^{XX} - (\tilde{\kappa}_{e+})^{YY}$	$10^{-32}$
$(\tilde{\kappa}_{e+})^{ZZ}$	$10^{-32}$
$(\tilde{\kappa}_{o-})^{XY}$	$10^{-32}$
$(\tilde{\kappa}_{o-})^{XZ}$	$10^{-32}$
$(\tilde{\kappa}_{o-})^{YZ}$	$10^{-32}$
$(\tilde{\kappa}_{o-})^{XX} - (\tilde{\kappa}_{o-})^{YY}$	$10^{-32}$
$(\tilde{\kappa}_{o-})^{ZZ}$	$10^{-32}$
$(\tilde{\kappa}_{e-})^{XY}$	$10^{-16}$
$(\tilde{\kappa}_{e-})^{XZ}$	$10^{-16}$
$(\tilde{\kappa}_{e-})^{YZ}$	$10^{-16}$
$(\tilde{\kappa}_{e-})^{XX} - (\tilde{\kappa}_{e-})^{YY}$	$10^{-17}$
$(\tilde{\kappa}_{e-})^{ZZ}$	$10^{-17}$
$(\tilde{\kappa}_{o+})^{XY}$	$10^{-13}$
$(\tilde{\kappa}_{o+})^{XZ}$	$10^{-13}$
$(\tilde{\kappa}_{o+})^{YZ}$	$10^{-13}$
$\tilde{\kappa}_{\text{tr}}$	$10^{-14}$
$k_{(V)00}^{(3)}$	$10^{-43}$ GeV
$k_{(V)10}^{(3)}$	$10^{-42}$ GeV
$\text{Re } k_{(V)11}^{(3)}$	$10^{-42}$ GeV
$\text{Im } k_{(V)11}^{(3)}$	$10^{-42}$ GeV

TABLE IV: Maximal sensitivities for the gravity sector

Coefficient	Sensitivity
$\alpha \bar{a}_T^e$	$10^{-11}$ GeV
$\alpha \bar{a}_X^e$	—
$\alpha \bar{a}_Y^e$	—
$\alpha \bar{a}_Z^e$	—
$\alpha \bar{a}_T^p$	$10^{-11}$ GeV
$\alpha \bar{a}_X^p$	—
$\alpha \bar{a}_Y^p$	—
$\alpha \bar{a}_Z^p$	—
$\alpha \bar{a}_T^n$	$10^{-11}$ GeV
$\alpha \bar{a}_X^n$	—
$\alpha \bar{a}_Y^n$	—
$\alpha \bar{a}_Z^n$	—
$\bar{s}^{XX} - \bar{s}^{YY}$	$10^{-9}$
$\bar{s}^{XX} + \bar{s}^{YY} - 2\bar{s}^{ZZ}$	$10^{-7}$
$\bar{s}^{XY}$	$10^{-9}$
$\bar{s}^{XZ}$	$10^{-9}$
$\bar{s}^{YZ}$	$10^{-9}$
$\bar{s}^{TX}$	$10^{-6}$
$\bar{s}^{TY}$	$10^{-6}$
$\bar{s}^{TZ}$	$10^{-5}$
$\bar{s}^{TT}$	—

TABLE V: Electron sector

Combination	Result	System	Ref.
$\tilde{b}_X$	$(-0.9 \pm 1.4) \times 10^{-31}$ GeV	Torsion pendulum	[18]
$\tilde{b}_Y$	$(-0.9 \pm 1.4) \times 10^{-31}$ GeV	"	[18]
$\tilde{b}_Z$	$(-0.3 \pm 4.4) \times 10^{-30}$ GeV	"	[18]
$\frac{1}{2}(\tilde{b}_T + \tilde{d}_- - 2\tilde{g}_c - 3\tilde{g}_T + 4\tilde{d}_+ - \tilde{d}_Q)$	$(0.9 \pm 2.2) \times 10^{-27}$ GeV	"	[18]
$\frac{1}{2}(2\tilde{g}_c - \tilde{g}_T - \tilde{b}_T + 4\tilde{d}_+ - \tilde{d}_- - \tilde{d}_Q)$ $+ \tan \eta(\tilde{d}_{YZ} - \tilde{H}_{XT})$	$(-0.8 \pm 2.0) \times 10^{-27}$ GeV	"	[18]
$\tilde{b}_X$	$(0.1 \pm 2.4) \times 10^{-31}$ GeV	"	[19]
$\tilde{b}_Y$	$(-1.7 \pm 2.5) \times 10^{-31}$ GeV	"	[19]
$\tilde{b}_Z$	$(-29 \pm 39) \times 10^{-31}$ GeV	"	[19]
$\tilde{b}_\perp$	$< 3.1 \times 10^{-29}$ GeV	"	[20]
$ \tilde{b}_Z $	$< 7.1 \times 10^{-28}$ GeV	"	[20]
$\tilde{b}_X$	$(2.8 \pm 6.1) \times 10^{-29}$ GeV	K/He magnetometer	[21]
$\tilde{b}_Y$	$(6.8 \pm 6.1) \times 10^{-29}$ GeV	"	[21]
$r_e$	$< 3.2 \times 10^{-24}$	Hg/Cs comparison	[22]
$ \vec{b} $	$< 20$ radians/s	Penning trap	[23]
$r_{\omega_a^-, \text{diurnal}}$	$< 1.6 \times 10^{-21}$	"	[24]
$ \tilde{b}_J $ ( $J = X, Y$ )	$< 10^{-27}$ GeV	Hg/Cs comparison	[25]*
$0.83c_{(TX)} + 0.51c_{(TY)} + 0.22c_{(TZ)}$	$(4 \pm 8) \times 10^{-11}$	1S-2S transition	[26]*
$c_{XX} - c_{YY}$	$(-2.9 \pm 6.3) \times 10^{-16}$	Optical, microwave resonators	[27]*
$\frac{1}{2}c_{(XY)}$	$(2.1 \pm 0.9) \times 10^{-16}$	"	[27]*
$\frac{1}{2}c_{(XZ)}$	$(-1.5 \pm 0.9) \times 10^{-16}$	"	[27]*
$\frac{1}{2}c_{(YZ)}$	$(-0.5 \pm 1.2) \times 10^{-16}$	"	[27]*
$c_{XX} + c_{YY} - 2c_{ZZ}$	$(-106 \pm 147) \times 10^{-16}$	"	[27]*
$\lambda^{ZZ}$	$(13.3 \pm 9.8) \times 10^{-16}$	"	[27]*
$c_{(YZ)}$	$(2.1 \pm 4.6) \times 10^{-16}$	"	[28]*
$c_{(XZ)}$	$(-1.6 \pm 6.3) \times 10^{-16}$	"	[28]*
$c_{(XY)}$	$(7.6 \pm 3.5) \times 10^{-16}$	"	[28]*
$c_{XX} - c_{YY}$	$(1.15 \pm 0.64) \times 10^{-15}$	"	[28]*
$ c_{XX} + c_{YY} - 2c_{ZZ} - 0.25(\tilde{\kappa}_{e-})^{ZZ} $	$< 10^{-12}$	"	[28]*
$ \frac{1}{2}c_{(XY)} $	$< 8 \times 10^{-15}$	Optical resonators	[29]*
$ c_{XX} - c_{YY} $	$< 1.6 \times 10^{-15}$	"	[29]*
$ c_{XX} + c_{YY} - 2c_{ZZ} $	$< 10^{-5}$	Heavy-ion storage ring	[30]*
$ c_{(TX)} ,  c_{(TY)} ,  c_{(TZ)} $	$< 10^{-2}$	"	[30]*

TABLE V: Electron sector (continued)

Combination	Result	System	Ref.
$c_{XX}$	$(-3 \text{ to } 5) \times 10^{-15}$	Astrophysics	[31]*
$c_{YY}$	$(-0.7 \text{ to } 2.5) \times 10^{-15}$	"	[31]*
$c_{ZZ}$	$(-1.6 \text{ to } 2.5) \times 10^{-15}$	"	[31]*
$c_{(YZ)}$	$(-2.5 \text{ to } 1.8) \times 10^{-15}$	"	[31]*
$c_{0X}$	$(-7 \text{ to } 4) \times 10^{-15}$	"	[31]*
$c_{0Y}$	$(-0.5 \text{ to } 1.5) \times 10^{-15}$	"	[31]*
$c_{0Z}$	$(-4 \text{ to } 2) \times 10^{-17}$	"	[31]*
$ 0.05c_{XX} + 0.55c_{YY} + 0.41c_{ZZ}$ $+0.16c_{(XY)} - 0.14c_{(XZ)} - 0.47c_{(YZ)}$ $+0.22c_{(0X)} + 0.74c_{(0Y)} - 0.64c_{(0Z)} + c_{00} $	$< 1.3 \times 10^{-15}$	"	[32]*
$ 0.58c_{XX} + 0.04c_{YY} + 0.38c_{ZZ}$ $-0.14c_{(XY)} - 0.47c_{(XZ)} + 0.12c_{(YZ)}$ $+0.76c_{(0X)} - 0.19c_{(0Y)} - 0.62c_{(0Z)} + c_{00} $	$< 2.5 \times 10^{-15}$	"	[32]*
$c_{TT} \equiv -\delta$	$(-13 \text{ to } 2) \times 10^{-16}$	"	[33]*
$\tilde{d}_{XY} - \tilde{H}_{ZT} + \tan \eta \tilde{H}_{YT}$	$(0.1 \pm 1.8) \times 10^{-27} \text{ GeV}$	Torsion pendulum	[18]
$\tilde{H}_{ZT}$	$(-4.1 \pm 2.4) \times 10^{-27} \text{ GeV}$	"	[18]
$\tilde{H}_{YT} - \tilde{d}_{ZX}$	$(-4.9 \pm 8.9) \times 10^{-27} \text{ GeV}$	"	[18]
$-\tilde{H}_{XT} + \tan \eta (\tilde{g}_T - 2\tilde{d}_+ + \tilde{d}_Q)$	$(1.1 \pm 9.2) \times 10^{-27} \text{ GeV}$	"	[18]
$ d_{XX} $	$< 2 \times 10^{-14}$	Astrophysics	[34]*
$ d_{YY} ,  d_{ZZ} $	$< 3 \times 10^{-15}$	"	[34]*
$ d_{(XY)} $	$< 2 \times 10^{-15}$	"	[34]*
$ d_{(XZ)} $	$< 2 \times 10^{-14}$	"	[34]*
$ d_{(YZ)} $	$< 7 \times 10^{-15}$	"	[34]*
$ d_{TX} $	$< 5 \times 10^{-14}$	"	[34]*
$ d_{TY} $	$< 5 \times 10^{-15}$	"	[34]*
$ d_{TZ} $	$< 8 \times 10^{-17}$	"	[34]*
$ \tilde{d}_J ,  \tilde{g}_{D,J}  \text{ } (J = X, Y)$	$< 10^{-22} \text{ GeV}$	Hg/Cs comparison	[25]*

TABLE VI: Proton sector

Combination	Result	System	Ref.
$\tilde{b}_X$	$(6.0 \pm 1.3) \times 10^{-31}$ GeV	K/He magnetometer	[21]
$\tilde{b}_Y$	$(1.5 \pm 1.2) \times 10^{-31}$ GeV	"	[21]
$\sqrt{(\tilde{b}_X^e + \tilde{b}_X^p)^2 + (\tilde{b}_Y^e + \tilde{b}_Y^p)^2}$	$(3 \pm 2) \times 10^{-27}$ GeV	H maser	[35]
$ \tilde{b}_J $ ( $J = X, Y$ )	$< 2 \times 10^{-27}$ GeV	"	[36]
$ \tilde{b}_J $ ( $J = X, Y$ )	$< 10^{-27}$ GeV	Hg/Cs comparison	[25]*
$\tilde{c}_Q$	$(-0.3 \pm 2.2) \times 10^{-22}$ GeV	Cs fountain	[37]
$\tilde{c}_-$	$(-1.8 \pm 2.8) \times 10^{-25}$ GeV	"	[37]
$\tilde{c}_X$	$(0.6 \pm 1.2) \times 10^{-25}$ GeV	"	[37]
$\tilde{c}_Y$	$(-1.9 \pm 1.2) \times 10^{-25}$ GeV	"	[37]
$\tilde{c}_Z$	$(-1.4 \pm 2.8) \times 10^{-25}$ GeV	"	[37]
$\tilde{c}_{TX}$	$(-2.7 \pm 3.0) \times 10^{-21}$ GeV	"	[37]
$\tilde{c}_{TY}$	$(-0.2 \pm 3.0) \times 10^{-21}$ GeV	"	[37]
$\tilde{c}_{TZ}$	$(-0.4 \pm 2.0) \times 10^{-21}$ GeV	"	[37]
$ c_{XX} + c_{YY} - 2c_{ZZ} $	$< 10^{-11}$	Doppler shift	[30]*
$ c_{(TX)} ,  c_{(TY)} ,  c_{(TZ)} $	$< 10^{-8}$	"	[30]*
$r_{\omega_c}^{H^-}$	$< 4 \times 10^{-26}$	Penning trap	[38]
$ \tilde{d}_J ,  \tilde{g}_{D,J} $ ( $J = X, Y$ )	$< 10^{-25}$ GeV	Hg/Cs comparison	[25]*

TABLE VII: Neutron sector

Combination	Result	System	Ref.
$b_{\perp}$	$< 2 \times 10^{-29}$ GeV	Ultra-cold neutrons	[39]
$-4.2b_i^{(n)} + 0.7b_i^{(p)}$	$2\pi(53 \pm 45)$ nHz	Xe/He maser	[40]
$ b_J - \frac{1}{2}\epsilon_{JKL}H_{KL} , (J = X, Y)$	$< 10^{-28}$ GeV	Maser/magnetometer	[41]*
$\tilde{b}_X$	$(-3.7 \pm 8.1) \times 10^{-32}$ GeV	K/He magnetometer	[21]
$\tilde{b}_Y$	$(-9.0 \pm 7.5) \times 10^{-32}$ GeV	"	[21]
$\tilde{b}_Y - 0.0034\tilde{d}_Y + 0.0034\tilde{g}_{DY}$	$(8.0 \pm 9.5) \times 10^{-32}$ GeV	Xe/He maser	[42]
$-\tilde{b}_X - 0.0034\tilde{d}_X - 0.0034\tilde{g}_{DX}$	$(2.2 \pm 7.9) \times 10^{-32}$ GeV	"	[42]
$-\cos\eta(\frac{1}{2}\tilde{b}_T + \frac{1}{2}\tilde{d}_- - \tilde{g}_c - \frac{1}{2}\tilde{g}_T)$	$(-1.1 \pm 1.0) \times 10^{-27}$ GeV	"	[42]
$-\cos\eta(\tilde{g}_T - 2\tilde{d}_+ + \frac{1}{2}\tilde{d}_Q)$			
$+\sin\eta(\tilde{d}_{YZ} - \tilde{H}_{XT})$			
$-\tilde{H}_{ZT}$	$(0.2 \pm 1.8) \times 10^{-27}$ GeV	"	[42]
$(\frac{1}{2}\tilde{b}_T + \frac{1}{2}\tilde{d}_- - \tilde{g}_c - \frac{1}{2}\tilde{g}_T)$	$(-1.8 \pm 1.9) \times 10^{-27}$ GeV	"	[42]
$-(\tilde{g}_T - 2\tilde{d}_+ + \frac{1}{2}\tilde{d}_Q)$			
$\cos\eta(\tilde{H}_{ZT} - \tilde{d}_{XY}) - \sin\eta\tilde{H}_{YT}$	$(-1.1 \pm 0.8) \times 10^{-27}$ GeV	"	[42]
$\sqrt{(\tilde{b}_X)^2 + (\tilde{b}_Y)^2}$	$(6.4 \pm 5.4) \times 10^{-32}$ GeV	"	[43]
$r_n$	$< 1.5 \times 10^{-30}$	Hg/Cs comparison	[22]
$ \tilde{b}_J  (J = X, Y)$	$< 10^{-30}$ GeV	"	[25]*
$\frac{1}{4} c_Q ,  c_{(TJ)} $	$< 5 \times 10^{-14}$	Astrophysics	[44]*
$\min( c_{11} - c_{22} ,  c_{11} - c_{33} ,  c_{22} - c_{33} )$	$< 1.7 \times 10^{-8}$	Pulsar timing	[45]*
$ \tilde{c}_J  (J = X, Y)$	$< 10^{-25}$ GeV	Be/H comparison	[25]*
$ \tilde{c}_- ,  \tilde{c}_Z $	$< 10^{-27}$ GeV	Hg/Hg & Ne/He comparison	[25]*
$ md_{JT} - \frac{1}{2}\epsilon_{JKL}mg_{KLT} , (J = X, Y)$	$< 10^{-28}$ GeV	Maser/magnetometer	[41]*
$\frac{1}{2} d_{(XZ)} ,  d_{(TZ)} $	$< 5 \times 10^{-14}$	Astrophysics	[44]*
$ \tilde{d}_J ,  \tilde{g}_{D,J}  (J = X, Y)$	$< 10^{-28}$ GeV	Hg/Cs comparison	[25]*

TABLE VIII: Photon sector

Combination	Result	System	Ref.
$(\tilde{\kappa}_{e-})^{XY}$	$(-0.31 \pm 0.73) \times 10^{-17}$	Rotating optical resonators	[46]
”	$(0.0 \pm 1.0 \pm 0.3) \times 10^{-17}$	”	[47]
”	$(-0.1 \pm 0.6) \times 10^{-17}$	”	[48]
”	$(-7.7 \pm 4.0) \times 10^{-16}$	Optical, microwave resonators	[27]*
”	$(2.9 \pm 2.3) \times 10^{-16}$	Rotating microwave resonators	[49]
”	$(-3.1 \pm 2.5) \times 10^{-16}$	Rotating optical resonators	[50]
”	$(-0.63 \pm 0.43) \times 10^{-15}$	Rotating microwave resonators	[51]
”	$(-1.7 \pm 1.6) \times 10^{-15}$	Optical, microwave resonators	[28]*
”	$(-5.7 \pm 2.3) \times 10^{-15}$	Microwave resonator, maser	[52]
”	$(1.7 \pm 2.6) \times 10^{-15}$	Optical resonators	[53]
”	$(1.4 \pm 1.4) \times 10^{-13}$	Microwave resonators	[54]
$(\tilde{\kappa}_{e-})^{XZ}$	$(0.54 \pm 0.70) \times 10^{-17}$	Rotating optical resonators	[46]
”	$(0.4 \pm 1.5 \pm 0.1) \times 10^{-17}$	”	[47]
”	$(-2.0 \pm 0.9) \times 10^{-17}$	”	[48]
”	$(-10.3 \pm 3.9) \times 10^{-16}$	Optical, microwave resonators	[27]*
”	$(-6.9 \pm 2.2) \times 10^{-16}$	Rotating microwave resonators	[49]
”	$(5.7 \pm 4.9) \times 10^{-16}$	Rotating optical resonators	[50]
”	$(0.19 \pm 0.37) \times 10^{-15}$	Rotating microwave resonators	[51]
”	$(-4.0 \pm 3.3) \times 10^{-15}$	Optical, microwave resonators	[28]*
”	$(-3.2 \pm 1.3) \times 10^{-15}$	Microwave resonator, maser	[52]
”	$(-6.3 \pm 12.4) \times 10^{-15}$	Optical resonators	[53]
”	$(-3.5 \pm 4.3) \times 10^{-13}$	Microwave resonators	[54]
$(\tilde{\kappa}_{e-})^{YZ}$	$(-0.97 \pm 0.74) \times 10^{-17}$	Rotating optical resonators	[46]
”	$(-0.6 \pm 1.4 \pm 0.5) \times 10^{-17}$	”	[47]
”	$(-0.3 \pm 1.4) \times 10^{-17}$	”	[48]
”	$(0.9 \pm 4.2) \times 10^{-16}$	Optical, microwave resonators	[27]*
”	$(2.1 \pm 2.1) \times 10^{-16}$	Rotating microwave resonators	[49]
”	$(-1.5 \pm 4.4) \times 10^{-16}$	Rotating optical resonators	[50]
”	$(-0.45 \pm 0.37) \times 10^{-15}$	Rotating microwave resonators	[51]
”	$(0.52 \pm 2.52) \times 10^{-15}$	Optical, microwave resonators	[28]*
”	$(-0.5 \pm 1.3) \times 10^{-15}$	Microwave resonator, maser	[52]
”	$(3.6 \pm 9.0) \times 10^{-15}$	Optical resonators	[53]
”	$(1.7 \pm 3.6) \times 10^{-13}$	Microwave resonators	[54]

TABLE VIII: Photon sector (continued)

Combination	Result	System	Ref.
$(\tilde{\kappa}_{e-})^{XX} - (\tilde{\kappa}_{e-})^{YY}$	$(0.80 \pm 1.27) \times 10^{-17}$	Rotating optical resonators	[46]
”	$(0.8 \pm 2.0 \pm 0.3) \times 10^{-17}$	”	[47]
”	$(-2.0 \pm 1.7) \times 10^{-17}$	”	[48]
”	$(-12 \pm 16) \times 10^{-16}$	Optical, microwave resonators	[27]*
”	$(-5.0 \pm 4.7) \times 10^{-16}$	Rotating microwave resonators	[49]
”	$(5.4 \pm 4.8) \times 10^{-16}$	Rotating optical resonators	[50]
”	$(-1.3 \pm 0.9) \times 10^{-15}$	Rotating microwave resonators	[51]
”	$(2.8 \pm 3.3) \times 10^{-15}$	Optical, microwave resonators	[28]*
”	$(-3.2 \pm 4.6) \times 10^{-15}$	Microwave resonator, maser	[52]
”	$(8.9 \pm 4.9) \times 10^{-15}$	Optical resonators	[53]
”	$(-1.0 \pm 2.1) \times 10^{-13}$	Microwave resonators	[54]
$(\tilde{\kappa}_{e-})^{ZZ}$	$(-0.04 \pm 1.73) \times 10^{-17}$	Rotating optical resonators	[46]
”	$(1.6 \pm 2.4 \pm 1.1) \times 10^{-17}$	”	[47]
”	$(-0.2 \pm 3.1) \times 10^{-17}$	”	[48]
”	$(223 \pm 290) \times 10^{-16}$	Optical, microwave resonators	[27]*
”	$(143 \pm 179) \times 10^{-16}$	Rotating microwave resonators	[49]
”	$(-1.9 \pm 5.2) \times 10^{-15}$	Rotating optical resonators	[50]
”	$(21 \pm 57) \times 10^{-15}$	Rotating microwave resonators	[51]
”	$(-2.9 \pm 2.2) \times 10^{-14}$	Optical resonators	[55]
$ (\tilde{\kappa}_{e-})^{(kl)} $	$< 4 \times 10^{-18}$	Astrophysics	[56]*

TABLE VIII: Photon sector (continued)

Combination	Result	System	Ref.
$\beta_{\oplus}(\tilde{\kappa}_{o+})^{XY}$	$(-0.14 \pm 0.78) \times 10^{-17}$	Rotating optical resonators	[46]
$(\tilde{\kappa}_{o+})^{XY}$	$(1.5 \pm 1.5 \pm 0.2) \times 10^{-13}$	"	[47]
$\beta_{\oplus}(\tilde{\kappa}_{o+})^{XY}$	$(-2.5 \pm 2.5) \times 10^{-17}$	"	[48]
$(\tilde{\kappa}_{o+})^{XY}$	$(1.7 \pm 2.0) \times 10^{-12}$	Optical, microwave resonators	[27]*
"	$(-0.9 \pm 2.6) \times 10^{-12}$	Rotating microwave resonators	[49]
"	$(-2.5 \pm 5.1) \times 10^{-12}$	Rotating optical resonators	[50]
"	$(0.20 \pm 0.21) \times 10^{-11}$	Rotating microwave resonators	[51]
"	$(-1.8 \pm 1.5) \times 10^{-11}$	Microwave resonator, maser	[52]
"	$(14 \pm 14) \times 10^{-11}$	Optical resonators	[53]
$\beta_{\oplus}(\tilde{\kappa}_{o+})^{XZ}$	$(-0.45 \pm 0.62) \times 10^{-17}$	Rotating optical resonators	[46]
$(\tilde{\kappa}_{o+})^{XZ}$	$(-0.1 \pm 1.0 \pm 0.2) \times 10^{-13}$	"	[47]
$\beta_{\oplus}(\tilde{\kappa}_{o+})^{XZ}$	$(1.5 \pm 1.7) \times 10^{-17}$	"	[48]
$(\tilde{\kappa}_{o+})^{XZ}$	$(-3.1 \pm 2.3) \times 10^{-12}$	Optical, microwave resonators	[27]*
"	$(-4.4 \pm 2.5) \times 10^{-12}$	Rotating microwave resonators	[49]
"	$(-3.6 \pm 2.7) \times 10^{-12}$	Rotating optical resonators	[50]
"	$(-0.91 \pm 0.46) \times 10^{-11}$	Rotating microwave resonators	[51]
"	$(-1.4 \pm 2.3) \times 10^{-11}$	Microwave resonator, maser	[52]
"	$(-1.2 \pm 2.6) \times 10^{-11}$	Optical resonators	[53]
$\beta_{\oplus}(\tilde{\kappa}_{o+})^{YZ}$	$(-0.34 \pm 0.61) \times 10^{-17}$	Rotating optical resonators	[46]
$(\tilde{\kappa}_{o+})^{YZ}$	$(-0.1 \pm 1.0 \pm 0.4) \times 10^{-13}$	"	[47]
$\beta_{\oplus}(\tilde{\kappa}_{o+})^{YZ}$	$(-1.0 \pm 1.5) \times 10^{-17}$	"	[48]
$(\tilde{\kappa}_{o+})^{YZ}$	$(-2.8 \pm 2.2) \times 10^{-12}$	Optical, microwave resonators	[27]*
"	$(-3.2 \pm 2.3) \times 10^{-12}$	Rotating microwave resonators	[49]
"	$(2.9 \pm 2.8) \times 10^{-12}$	Rotating optical resonators	[50]
"	$(0.44 \pm 0.46) \times 10^{-11}$	Rotating microwave resonators	[51]
"	$(2.7 \pm 2.2) \times 10^{-11}$	Microwave resonator, maser	[52]
"	$(0.1 \pm 2.7) \times 10^{-11}$	Optical resonators	[53]
$(\tilde{\kappa}_{o+})^{YX} - 0.432(\tilde{\kappa}_{o+})^{ZX}$	$(4.0 \pm 8.4) \times 10^{-9}$	Microwave resonators	[54]
$(\tilde{\kappa}_{o+})^{XY} - 0.209(\tilde{\kappa}_{o+})^{YZ}$	$(4.0 \pm 4.9) \times 10^{-9}$	"	[54]
$(\tilde{\kappa}_{o+})^{XZ} - 0.484(\tilde{\kappa}_{o+})^{YZ}$	$(1.6 \pm 1.7) \times 10^{-9}$	"	[54]
$(\tilde{\kappa}_{o+})^{YZ} + 0.484(\tilde{\kappa}_{o+})^{XZ}$	$(0.6 \pm 1.9) \times 10^{-9}$	"	[54]
$ (\tilde{\kappa}_{o+})^{(ij)} $	$< 2 \times 10^{-18}$	Astrophysics	[56]*

TABLE VIII: Photon sector (continued)

Combination	Result	System	Ref.
$\tilde{\kappa}_{\text{tr}}$	$(-0.3 \pm 3) \times 10^{-7}$	Microwave interferometer	[57]
$ \tilde{\kappa}_{\text{tr}} - \frac{4}{3}c_{00}^e $	$< 5 \times 10^{-15}$	Collider physics	[58]*
$\tilde{\kappa}_{\text{tr}} - \frac{4}{3}c_{00}^e$	$(-5.8 \text{ to } 12) \times 10^{-12}$	"	[59]*
$\tilde{\kappa}_{\text{tr}} - \frac{4}{3}c_{00}^p$	$< 6 \times 10^{-20}$	Astrophysics	[60]*
$-\left[\tilde{\kappa}_{\text{tr}} - \frac{4}{3}c_{00}^e\right]$	$< 9 \times 10^{-16}$	"	[60]*
$\tilde{\kappa}_{\text{tr}}$	$< 1.4 \times 10^{-19}$	"	[56]*
$ \tilde{\kappa}_{\text{tr}} $	$< 8.4 \times 10^{-8}$	Optical atomic clocks	[61]
"	$< 2.2 \times 10^{-7}$	Heavy-ion storage ring	[62]*
"	$< 2 \times 10^{-14}$	Astrophysics	[63]*
"	$< 3 \times 10^{-8}$	$g_e - 2$	[63]*
"	$< 1.6 \times 10^{-5}$	Sagnac interferometer	[64]*
$ \sum_{jm} {}_2Y_{jm}(98.2^\circ, 182.1^\circ)(k_{(E)jm}^{(4)} + ik_{(B)jm}^{(4)}) $	$\lesssim 10^{-37}$	Astrophysical birefringence	[13]*
$ \sum_{jm} {}_2Y_{jm}(87.3^\circ, 37.3^\circ)(k_{(E)jm}^{(4)} + ik_{(B)jm}^{(4)}) $	$\lesssim 10^{-37}$	"	[13]*
$k_{(E)20}^{(4)}$	$\pm(17_{-9}^{+7}) \times 10^{-31}$	CMB polarization	[10]*
$k_{(B)20}^{(4)}$	$\pm(17_{-9}^{+7}) \times 10^{-31}$	"	[10]*
$\sqrt{\sum_m ( k_{(E)2m}^{(4)} ^2 +  k_{(B)2m}^{(4)} ^2)}$	$< 5 \times 10^{-32}$	Astrophysical birefringence	[9]*, [13]*
$ k^a $ for some $a$	$< 2 \times 10^{-37}$	"	[65]*
$ k^a $ for $a = 1, \dots, 10$	$< 2 \times 10^{-32}$	"	[9]*
$ k_{(V)10}^{(3)} $	$< 16 \times 10^{-21}$ GeV	Schumann resonances	[66]*
$ k_{(V)11}^{(3)} $	$< 12 \times 10^{-21}$ GeV	"	[66]*
$ \mathbf{k}_{\text{AF}}^{(3)}  \equiv \left(6 k_{(V)11}^{(3)} ^2 + 3 k_{(V)10}^{(3)} ^2\right)^{1/2} / \sqrt{4\pi}$	$(10_{-8}^{+4}) \times 10^{-43}$ GeV	CMB polarization	[12]*
$ \mathbf{k}_{\text{AF}}^{(3)} $	$(15 \pm 6) \times 10^{-43}$ GeV	"	[10]*, [13]*
$k_{(V)10}^{(3)}$	$\pm(3 \pm 1) \times 10^{-42}$ GeV	"	[10]*
$\text{Re } k_{(V)11}^{(3)}$	$\pm(21_{-9}^{+7}) \times 10^{-43}$ GeV	"	[10]*
$ \sum_{jm} {}_0Y_{jm}k_{(V)jm}^{(3)} $	$< 6 \times 10^{-43}$ GeV	Astrophysical birefringence	[67]*, [13]*
$ k_{(V)00}^{(3)} $	$< 14 \times 10^{-21}$ GeV	Schumann resonances	[66]*
$k_{(V)00}^{(3)}$	$(-1.4 \pm 0.9 \pm 0.5) \times 10^{-43}$ GeV	CMB polarization	[68]
"	$(2.3 \pm 5.4) \times 10^{-43}$ GeV	"	[12]*
"	$< 2.5 \times 10^{-43}$ GeV	"	[69]*, [12]*
"	$(1.2 \pm 2.2) \times 10^{-43}$ GeV	"	[70], [12]*
"	$(12 \pm 7) \times 10^{-43}$ GeV	"	[10]*
"	$(2.6 \pm 1.9) \times 10^{-43}$ GeV	"	[71]*, [12]*
"	$(2.5 \pm 3.0) \times 10^{-43}$ GeV	"	[72]*, [12]*
"	$(6.0 \pm 4.0) \times 10^{-43}$ GeV	"	[73]*, [10]*
"	$< 2 \times 10^{-42}$ GeV	Astrophysical birefringence	[67]*, [13]*

TABLE IX: Charged-lepton sector

Combination	Result	System	Ref.
$b_Z^\mu$	$-(1.0 \pm 1.1) \times 10^{-23}$ GeV	BNL $g_\mu - 2$	[74]
$\sqrt{(\tilde{b}_X^{\mu+})^2 + (\tilde{b}_Y^{\mu+})^2}$	$< 1.4 \times 10^{-24}$ GeV	"	[74]
$\sqrt{(\tilde{b}_X^{\mu-})^2 + (\tilde{b}_Y^{\mu-})^2}$	$< 2.6 \times 10^{-24}$ GeV	"	[74]
$\sqrt{(\tilde{b}_X^\mu)^2 + (\tilde{b}_Y^\mu)^2}$	$< 2 \times 10^{-23}$ GeV	Muonium spectroscopy	[75]
$b_Z^\mu - 1.19(m_\mu d_{Z0}^\mu + H_{XY}^\mu)$	$(-1.4 \pm 1.0) \times 10^{-22}$ GeV	BNL, CERN $g_\mu - 2$ data	[76]
$b_Z^\mu$	$(-2.3 \pm 1.4) \times 10^{-22}$ GeV	CERN $g_\mu - 2$ data	[77]*, [76]
$m_\mu d_{Z0}^\mu + H_{XY}^\mu$	$(1.8 \pm 6.0) \times 10^{-23}$ GeV	BNL $g_\mu - 2$	[74]
$ c^\mu $	$< 10^{-11}$	Astrophysics	[32]*
$ c^\tau $	$< 10^{-8}$	"	[32]*

TABLE X: Neutrino sector

Combination	Result	System	Ref.
$(c_L^{\nu e})_{00}$	$< 2 \times 10^{-11}$	Cosmic rays	[78]*
$ a_L^X ,  a_L^Y $	$< 3.0 \times 10^{-20}$ GeV	Accelerator	[79]
$ c_L^{TX} ,  c_L^{TY} $	$< 9 \times 10^{-23}$	"	[79]
$ c_L^{XX} $	$< 5.6 \times 10^{-21}$	"	[79]
$ c_L^{YY} $	$< 5.5 \times 10^{-21}$	"	[79]
$ c_L^{XY} $	$< 2.7 \times 10^{-21}$	"	[79]
$ c_L^{YZ} $	$< 1.2 \times 10^{-21}$	"	[79]
$ c_L^{XZ} $	$< 1.3 \times 10^{-21}$	"	[79]
$ (C)_{\bar{e}\bar{\mu}} ^2$	$(10.7 \pm 2.6 \pm 1.3) \times (10^{-19} \text{ GeV})^2$	"	[80]
$ (C)_{\bar{e}\bar{\mu}} ^2 + \frac{1}{2} (\mathcal{A}_s)_{\bar{e}\bar{\mu}} ^2 + \frac{1}{2} (\mathcal{A}_c)_{\bar{e}\bar{\mu}} ^2$	$(9.9 \pm 2.3 \pm 1.4) \times (10^{-19} \text{ GeV})^2$	"	[80]
$ (C)_{\bar{e}\bar{\mu}} ^2 + \frac{1}{2} (\mathcal{A}_s)_{\bar{e}\bar{\mu}} ^2 + \frac{1}{2} (\mathcal{A}_c)_{\bar{e}\bar{\mu}} ^2$ $+ \frac{1}{2} (\mathcal{B}_s)_{\bar{e}\bar{\mu}} ^2 + \frac{1}{2} (\mathcal{B}_c)_{\bar{e}\bar{\mu}} ^2$	$(10.5 \pm 2.4 \pm 1.4) \times (10^{-19} \text{ GeV})^2$	"	[80]
$a \cos \rho$	excluded	Multiple	[81]*
$a \sin \rho \hat{n}$	excluded	"	[81]*
$c$	excluded	"	[81]*
$b$	$< 1.6 \times 10^{-23}$ GeV	Atmospheric	[82]
$c$	$< 1.4 \times 10^{-26}$	"	[82]
$\check{a}/\check{c}$	$< 5$ GeV	"	[82]

TABLE XI: Meson sector

Combination	Result	System	Ref.
$\Delta a_X^K$	$(-6.3 \pm 6.0) \times 10^{-18}$ GeV	$K$ oscillations	[83]
$\Delta a_Y^K$	$(2.8 \pm 5.9) \times 10^{-18}$ GeV	"	[83]
$\Delta a_Z^K$	$(2.4 \pm 9.7) \times 10^{-18}$ GeV	"	[83]
$\Delta a_0^K$	$(0.4 \pm 1.8) \times 10^{-17}$ GeV	"	[83], [84]
$\Delta a_Z^K$	$(-1 \pm 4) \times 10^{-17}$ GeV	"	[84]
$ \Delta a_1^K $	$< 9.2 \times 10^{-22}$ GeV	"	[85]
$ \Delta a_2^K $	$< 9.2 \times 10^{-22}$ GeV	"	[85]
$ \Delta a_0^K - 0.6\Delta a_{\parallel}^K $	$< 10^{-20} m_K$	"	[86]*
$N^D(\Delta a_0^D - 0.6\Delta a_Z^D)$	$(-2.8 \text{ to } 4.8) \times 10^{-16}$ GeV	$D$ oscillations	[87]
$N^D \Delta a_X^D$	$(-7 \text{ to } 3.8) \times 10^{-16}$ GeV	"	[87]
$N^D \Delta a_Y^D$	$(-7 \text{ to } 3.8) \times 10^{-16}$ GeV	"	[87]
$N^B(\Delta a_0^B - 0.30\Delta a_Z^B)$	$(-3.0 \pm 2.4) \times 10^{-15}$ GeV	$B_d$ oscillations	[88]
$N^B \Delta a_X$	$(-22 \pm 7) \times 10^{-15}$ GeV	"	[88]
$N^B \Delta a_Y$	$(-27 \text{ to } -4) \times 10^{-15}$ GeV	"	[88]
$N^B(\Delta a_0^B - 0.3\Delta a_Z^B)$	$(-5.2 \pm 4.0) \times 10^{-15}$ GeV	"	[89]
$N^B \sqrt{(\Delta a_X^B)^2 + (\Delta a_Y^B)^2}$	$(37 \pm 16) \times 10^{-15}$ GeV	"	[89]
$\delta^\pi$	$(-1.5 \text{ to } 200) \times 10^{-11}$	Astrophysics	[90]*
$ c^\pi $	$< 10^{-10}$	"	[32]*
$ c^K $	$< 10^{-9}$	"	[32]*
$ c^D $	$< 10^{-8}$	"	[32]*
$ c^{B_d} ,  c^{B_s} $	$< 10^{-7}$	"	[32]*

TABLE XII: Electroweak sector

Combination	Result	System	Ref.
$ (k_{\phi\phi}^A)_{\mu\nu} $	$< 3 \times 10^{-16}$	Cosmological birefringence	[91]*
$ (k_{\phi B})_{\mu\nu} $	$< 0.9 \times 10^{-16}$	"	[91]*
$ (k_{\phi W})_{\mu\nu} $	$< 1.7 \times 10^{-16}$	"	[91]*
$ (k_{\phi\phi}^S)_{XX} ,  (k_{\phi\phi}^S)_{YY} ,  (k_{\phi\phi}^S)_{ZZ} $	$< 10^{-27}$	Clock comparisons	[91]*
$ (k_{\phi\phi}^S)_{XY} $	$< 10^{-27}$	"	[91]*
$ (k_{\phi\phi}^S)_{XZ} ,  (k_{\phi\phi}^S)_{YZ} $	$< 10^{-25}$	"	[91]*
$ (k_{\phi\phi}^S)_{TT} $	$< 4 \times 10^{-13}$	$H^-$ ion, $\bar{p}$ comparison	[91]*
$ (k_\phi)_X ,  (k_\phi)_Y $	$< 10^{-31}$	Xe-He maser	[91]*
$ (k_\phi)_Z ,  (k_\phi)_T $	$< 2.8 \times 10^{-27}$	"	[91]*
$ k_W $	$< 10^{-5}$	Astrophysics	[32]*

TABLE XIII: Gluon sector

Combination	Result	System	Ref.
$ \tilde{\kappa}_{\text{tr}}^{\text{QCD}} $	$< 2 \times 10^{-13}$	Astrophysics [63]*	

TABLE XIV: Gravity sector

Combination	Result	System	Ref.
$ \alpha\bar{a}_T^c + \alpha\bar{a}_T^p - 0.8\alpha\bar{a}_T^n $	$< 1 \times 10^{-11}$ GeV	Torsion pendulum	[11]*
$\sigma^{XX} - \sigma^{YY}$	$(4.4 \pm 11) \times 10^{-9}$	Atom interferometry	[92]
$\sigma^{XY}$	$(0.2 \pm 3.9) \times 10^{-9}$	"	[92]
$\sigma^{XZ}$	$(-2.6 \pm 4.4) \times 10^{-9}$	"	[92]
$\sigma^{YZ}$	$(-0.3 \pm 4.5) \times 10^{-9}$	"	[92]
$\sigma^{TX}$	$(-3.1 \pm 5.1) \times 10^{-5}$	"	[92]
$\sigma^{TY}$	$(0.1 \pm 5.4) \times 10^{-5}$	"	[92]
$\sigma^{TZ}$	$(1.4 \pm 6.6) \times 10^{-5}$	"	[92]
$\sigma^{XX} - \sigma^{YY}$	$(-5.6 \pm 2.1) \times 10^{-9}$	"	[93]
$\sigma^{XY}$	$(-0.09 \pm 79) \times 10^{-9}$	"	[93]
$\sigma^{XZ}$	$(-13 \pm 37) \times 10^{-9}$	"	[93]
$\sigma^{YZ}$	$(-61 \pm 38) \times 10^{-9}$	"	[93]
$\sigma^{TX}$	$(5.4 \pm 4.5) \times 10^{-5}$	"	[93]
$\sigma^{TY}$	$(-2.0 \pm 4.4) \times 10^{-5}$	"	[93]
$\sigma^{TZ}$	$(1.1 \pm 26) \times 10^{-5}$	"	[93]
$\bar{s}^{XX} - \bar{s}^{YY}$	$(-1.2 \pm 1.6) \times 10^{-9}$	LLR & Atom interferometry	[94], [92]*
$\bar{s}^{XX} + \bar{s}^{YY} - 2\bar{s}^{ZZ}$	$(1.8 \pm 38) \times 10^{-9}$	"	[94], [92]*
$\bar{s}^{XY}$	$(-0.6 \pm 1.5) \times 10^{-9}$	"	[94], [92]*
$\bar{s}^{XZ}$	$(-2.7 \pm 1.4) \times 10^{-9}$	"	[94], [92]*
$\bar{s}^{YZ}$	$(0.6 \pm 1.4) \times 10^{-9}$	"	[94], [92]*
$\bar{s}^{TX}$	$(0.5 \pm 6.2) \times 10^{-7}$	"	[94], [92]*
$\bar{s}^{TY}$	$(0.1 \pm 1.3) \times 10^{-6}$	"	[94], [92]*
$\bar{s}^{TZ}$	$(-0.4 \pm 3.8) \times 10^{-6}$	"	[94], [92]*
$\bar{s}^{11} - \bar{s}^{22}$	$(1.3 \pm 0.9) \times 10^{-10}$	Lunar laser ranging (LLR)	[94]
$\bar{s}^{12}$	$(6.9 \pm 4.5) \times 10^{-11}$	"	[94]
$\bar{s}^{01}$	$(-0.8 \pm 1.1) \times 10^{-6}$	"	[94]
$\bar{s}^{02}$	$(-5.2 \pm 4.8) \times 10^{-7}$	"	[94]
$\bar{s}_{\Omega_{\oplus}^c}$	$(0.2 \pm 3.9) \times 10^{-7}$	"	[94]
$\bar{s}_{\Omega_{\oplus}^s}$	$(-1.3 \pm 4.1) \times 10^{-7}$	"	[94]
$ \bar{s}_{\text{Mercury}} $	$\leq 10^{-9}$	Perihelion precession	[95]*
$ \bar{s}_{\oplus} $	$\leq 10^{-8}$	"	[95]*
$ \bar{s}_{\text{SSP}} $	$\leq 10^{-13}$	Solar-spin precession	[95]*

TABLE XV: Nonminimal photon sector

Combination	Result	System	Ref.
$ \sum_{jm} {}_0Y_{jm}(98.2^\circ, 182.1^\circ)k_{(V)jm}^{(5)} $	$< 7 \times 10^{-33} \text{ GeV}^{-1}$	Astrophysical birefringence	[13]*
$ k_{(V)00}^{(5)} $	$< 2 \times 10^{-32} \text{ GeV}^{-1}$	"	[13]*
$ \sum_{jm} {}_0Y_{jm}(87.3^\circ, 37.3^\circ)k_{(V)jm}^{(5)} $	$< 4 \times 10^{-33} \text{ GeV}^{-1}$	"	[13]*
$ k_{(V)00}^{(5)} $	$< 1 \times 10^{-32} \text{ GeV}^{-1}$	"	[13]*
$k_{(V)00}^{(5)}$	$(3.2 \pm 2.1) \times 10^{-20} \text{ GeV}^{-1}$	CMB polarization	[96]*
"	$(3 \pm 2) \times 10^{-20} \text{ GeV}^{-1}$	"	[10]*
$k_{(V)10}^{(5)}$	$(8_{-3}^{+2}) \times 10^{-20} \text{ GeV}^{-1}$	"	[10]*
	$-(8_{-4}^{+3}) \times 10^{-20} \text{ GeV}^{-1}$	"	[10]*
$k_{(V)20}^{(5)}$	$-(10 \pm 3) \times 10^{-20} \text{ GeV}^{-1}$	"	[10]*
$k_{(V)30}^{(5)}$	$(8_{-4}^{+3}) \times 10^{-20} \text{ GeV}^{-1}$	"	[10]*
	$-(8 \pm 3) \times 10^{-20} \text{ GeV}^{-1}$	"	[10]*
$ \sum_{jm} {}_2Y_{jm}(98.2^\circ, 182.1^\circ)(k_{(E)jm}^{(6)} + ik_{(B)jm}^{(6)}) $	$\lesssim 10^{-29} \text{ GeV}^{-2}$	Astrophysical birefringence	[13]*
$ \sum_{jm} {}_2Y_{jm}(87.3^\circ, 37.3^\circ)(k_{(E)jm}^{(6)} + ik_{(B)jm}^{(6)}) $	$\lesssim 10^{-29} \text{ GeV}^{-2}$	"	[13]*
$\sum_{jm} {}_0Y_{jm}(147^\circ, 120^\circ)c_{(I)jm}^{(6)}$	$< 3.2 \times 10^{-20} \text{ GeV}^{-2}$	Astrophysical dispersion	[97], [13]*
$c_{(I)00}^{(6)}$	$< 1.1 \times 10^{-19} \text{ GeV}^{-2}$	"	[97], [13]*
$ \sum_{jm} {}_0Y_{jm}(330^\circ, -30^\circ)c_{(I)jm}^{(6)} $	$< 7.4 \times 10^{-22} \text{ GeV}^{-2}$	"	[98], [13]*
$ c_{(I)00}^{(6)} $	$< 2.6 \times 10^{-21} \text{ GeV}^{-2}$	"	[98], [13]*
$\sum_{jm} {}_0Y_{jm}(50.2^\circ, 253^\circ)c_{(I)jm}^{(6)}$	$3_{-2}^{+1} \times 10^{-22} \text{ GeV}^{-2}$	"	[99], [13]*
$c_{(I)00}^{(6)}$	$10_{-7}^{+4} \times 10^{-22} \text{ GeV}^{-2}$	"	[99], [13]*
$ \sum_{jm} {}_0Y_{jm}(99.7^\circ, 240^\circ)c_{(I)jm}^{(6)} $	$< 1 \times 10^{-16} \text{ GeV}^{-2}$	"	[100], [13]*
$ c_{(I)00}^{(6)} $	$< 4 \times 10^{-16} \text{ GeV}^{-2}$	"	[100], [13]*
$k_{(E)20}^{(6)}$	$\pm(11_{-5}^{+4}) \times 10^{-10} \text{ GeV}^{-2}$	CMB polarization	[10]*
$k_{(E)30}^{(6)}$	$\pm(11_{-6}^{+5}) \times 10^{-10} \text{ GeV}^{-2}$	"	[10]*
$k_{(E)40}^{(6)}$	$\pm(11_{-6}^{+5}) \times 10^{-10} \text{ GeV}^{-2}$	"	[10]*
$ \sum_{jm} {}_0Y_{jm}(98.2^\circ, 182.1^\circ)k_{(V)jm}^{(7)} $	$< 2 \times 10^{-24} \text{ GeV}^{-3}$	Astrophysical birefringence	[13]*
$ k_{(V)00}^{(7)} $	$< 7 \times 10^{-24} \text{ GeV}^{-3}$	"	[13]*
$ \sum_{jm} {}_0Y_{jm}(87.3^\circ, 37.3^\circ)k_{(V)jm}^{(7)} $	$< 5 \times 10^{-25} \text{ GeV}^{-3}$	"	[13]*
$ k_{(V)00}^{(7)} $	$< 2 \times 10^{-24} \text{ GeV}^{-3}$	"	[13]*
$ \sum_{jm} {}_2Y_{jm}(98.2^\circ, 182.1^\circ)(k_{(E)jm}^{(8)} + ik_{(B)jm}^{(8)}) $	$\lesssim 10^{-20} \text{ GeV}^{-4}$	"	[13]*
$ \sum_{jm} {}_2Y_{jm}(87.3^\circ, 37.3^\circ)(k_{(E)jm}^{(8)} + ik_{(B)jm}^{(8)}) $	$\lesssim 10^{-20} \text{ GeV}^{-4}$	"	[13]*
$\sum_{jm} {}_0Y_{jm}(147^\circ, 120^\circ)c_{(I)jm}^{(8)}$	$< 2.6 \times 10^{-23} \text{ GeV}^{-4}$	Astrophysical dispersion	[97], [13]*
$c_{(I)00}^{(8)}$	$< 9.2 \times 10^{-23} \text{ GeV}^{-4}$	"	[97], [13]*
$ \sum_{jm} {}_0Y_{jm}(99.7^\circ, 240^\circ)c_{(I)jm}^{(8)} $	$< 3 \times 10^{-13} \text{ GeV}^{-4}$	"	[100], [13]*
$ c_{(I)00}^{(8)} $	$< 9 \times 10^{-13} \text{ GeV}^{-4}$	"	[100], [13]*
$ \sum_{jm} {}_0Y_{jm}(98.2^\circ, 182.1^\circ)k_{(V)jm}^{(9)} $	$< 6 \times 10^{-16} \text{ GeV}^{-5}$	Astrophysical birefringence	[13]*
$ k_{(V)00}^{(9)} $	$< 2 \times 10^{-15} \text{ GeV}^{-5}$	"	[13]*
$ \sum_{jm} {}_0Y_{jm}(87.3^\circ, 37.3^\circ)k_{(V)jm}^{(9)} $	$< 1 \times 10^{-16} \text{ GeV}^{-5}$	"	[13]*
$ k_{(V)00}^{(9)} $	$< 4 \times 10^{-16} \text{ GeV}^{-5}$	"	[13]*

TABLE XVI: Lagrange density for the minimal QED extension in Riemann spacetime

Sector	Coeff.	#	Operator	Dim.	Factor	CPT	L.V.
Fermion	$m$		$\bar{\psi}\psi$	3	$-e$	+	
	$m_5$		$\bar{\psi}\gamma_5\psi$	3	$-ie$	+	
			$\bar{\psi}\gamma^a\overleftrightarrow{D}_\mu\psi$	4	$\frac{1}{2}iee^\mu{}_a$	+	
	$a_\mu$	4	$\bar{\psi}\gamma^a\psi$	3	$-ee^\mu{}_a$	-	✓
	$b_\mu$	4	$\bar{\psi}\gamma_5\gamma^a\psi$	3	$-ee^\mu{}_a$	-	✓
	$H_{\mu\nu}$	6	$\bar{\psi}\sigma^{ab}\psi$	3	$-ee^\mu{}_ae^\nu{}_b$	+	✓
	$c_{\lambda\nu}$	16	$\bar{\psi}\gamma^b\overleftrightarrow{D}_\mu\psi$	4	$-\frac{1}{2}iee^\mu{}_ae^{\nu a}e^\lambda{}_b$	+	✓
	$d_{\lambda\nu}$	16	$\bar{\psi}\gamma_5\gamma^b\overleftrightarrow{D}_\mu\psi$	4	$-\frac{1}{2}iee^\mu{}_ae^{\nu a}e^\lambda{}_b$	+	✓
	$e_\lambda$	4	$\bar{\psi}\overleftrightarrow{D}_\mu\psi$	4	$-\frac{1}{2}iee^\mu{}_ae^{\lambda a}$	-	✓
	$f_\lambda$	4	$\bar{\psi}\gamma_5\overleftrightarrow{D}_\mu\psi$	4	$-\frac{1}{2}iee^\mu{}_ae^{\lambda a}$	-	✓
	$g_{\lambda\kappa\nu}$	24	$\bar{\psi}\sigma^{bc}\overleftrightarrow{D}_\mu\psi$	4	$-\frac{1}{4}iee^\mu{}_ae^{\nu a}e^\lambda{}_be^\kappa{}_c$	-	✓
Photon			$F_{\mu\nu}F^{\mu\nu}$	4	$-\frac{1}{4}e$	+	
	$(k_{AF})^\kappa$	4	$A^\lambda F^{\mu\nu}$	3	$\frac{1}{2}e\epsilon_{\kappa\lambda\mu\nu}$	-	✓
	$(k_F)_{\kappa\lambda\mu\nu}$	19	$F^{\kappa\lambda}F^{\mu\nu}$	4	$-\frac{1}{4}e$	+	✓
Gravity			$R$	2	$e/2\kappa$	+	
	$\Lambda$		1	0	$-e/\kappa$	+	
	$u$	1	$R$	2	$-e/2\kappa$	+	
	$s^{\mu\nu}$	9	$R_{\mu\nu}$	2	$e/2\kappa$	+	✓
	$t^{\kappa\lambda\mu\nu}$	10	$R_{\kappa\lambda\mu\nu}$	2	$e/2\kappa$	+	✓

TABLE XVII: C, P, T properties of operators for Lorentz violation in QED

Coefficient	C	P	T	CP	CT	PT	CPT
$c_{TT}, c_{JK}, (k_F)_{TJTK}, (k_F)_{JKLM}$	+	+	+	+	+	+	+
$b_J, g_{JTL}, g_{JKT}, (k_{AF})_J$	+	+	-	+	-	-	-
$b_T, g_{JTT}, g_{JKL}, (k_{AF})_T$	+	-	+	-	+	-	-
$c_{TJ}, c_{JT}, (k_F)_{TJKL}$	+	-	-	-	-	+	+
$a_T, e_T, f_J$	-	+	+	-	-	+	-
$H_{JK}, d_{TJ}, d_{JT}$	-	+	-	-	+	-	+
$H_{TJ}, d_{TT}, d_{JK}$	-	-	+	+	-	-	+
$a_J, e_J, f_T$	-	-	-	+	+	+	-

TABLE XVIII: Definitions for the fermion sector of the minimal QED extension

Symbol	Combination	Components
$\tilde{b}_J$	$b_J - \frac{1}{2}\varepsilon_{JKL}H_{KL} - m(d_{JT} - \frac{1}{2}\varepsilon_{JKL}g_{KLT})$	3
$\tilde{b}_J^*$	$b_J + \frac{1}{2}\varepsilon_{JKL}H_{KL} + m(d_{JT} + \frac{1}{2}\varepsilon_{JKL}g_{KLT}),$	3
$\tilde{b}_T$	$b_T + mg_{XYZ}$	1
$\tilde{g}_T$	$b_T - m(g_{XYZ} - g_{YZX} - g_{ZXY})$	1
$\tilde{H}_{XT}$	$H_{XT} + m(d_{ZY} - g_{XTT} - g_{XYX})$	1
$\tilde{H}_{YT}$	$H_{YT} + m(d_{XZ} - g_{YTT} - g_{YZZ})$	1
$\tilde{H}_{ZT}$	$H_{ZT} + m(d_{YX} - g_{ZTT} - g_{ZXX})$	1
$\tilde{d}_\pm$	$m(d_{XX} \pm d_{YY})$	2
$\tilde{d}_Q$	$m(d_{XX} + d_{YY} - 2d_{ZZ} - g_{YZX} - g_{ZXY} + 2g_{XYZ})$	1
$\tilde{d}_J$	$m(d_{TJ} + \frac{1}{2}d_{JT}) - \frac{1}{4}\varepsilon_{JKL}H_{KL}$	3
$\tilde{d}_{YZ}$	$m(d_{YZ} + d_{ZY} - g_{XYX} + g_{XZZ})$	1
$\tilde{d}_{ZX}$	$m(d_{ZX} + d_{XZ} - g_{YZZ} + g_{YXX})$	1
$\tilde{d}_{XY}$	$m(d_{XY} + d_{YX} - g_{ZXX} + g_{ZYY})$	1
$\tilde{g}_c$	$m(g_{XYZ} - g_{ZXY})$	1
$\tilde{g}_-$	$m(g_{XTX} - g_{YTY})$	1
$\tilde{g}_Q$	$m(g_{XTX} + g_{YTY} - 2g_{ZTZ})$	1
$\tilde{g}_{TJ}$	$m \varepsilon_{JKL} g_{KTL}$	3
$\tilde{g}_{DJ}$	$-b_J + m\varepsilon_{JKL}(g_{KTL} + \frac{1}{2}g_{KLT})$	3
$\tilde{g}_{JK}$	$m(g_{JTT} + g_{JKK}), (\text{no } K \text{ sum, } J \neq K)$	6
$\tilde{c}_Q$	$m(c_{XX} + c_{YY} - 2c_{ZZ})$	1
$\tilde{c}_-$	$m(c_{XX} - c_{YY})$	1
$\tilde{c}_J$	$m \varepsilon_{JKL} c_{KL}$	3
$\tilde{c}_{TJ}$	$m(c_{TJ} + c_{JT})$	3
$\tilde{c}_{TT}$	$m c_{TT}$	1 <b>total: 44</b>

TABLE XIX: Definitions for the photon sector of the minimal QED extension

Symbol	Combination	Components
$(\tilde{\kappa}_{e+})^{JK}$	$-(k_F)^{TJTK} + \frac{1}{4}\epsilon^{JPQ}\epsilon^{KRS}(k_F)^{PQRS}$	5
$(\tilde{\kappa}_{o-})^{JK}$	$\frac{1}{2}\epsilon^{K PQ}(k_F)^{TJPQ} + \frac{1}{2}\epsilon^{JPQ}(k_F)^{TKPQ}$	5
$(\tilde{\kappa}_{e-})^{JK}$	$-(k_F)^{TJTK} - \frac{1}{4}\epsilon^{JPQ}\epsilon^{KRS}(k_F)^{PQRS} + \frac{2}{3}(k_F)^{TLTL}\delta^{JK}$	5
$(\tilde{\kappa}_{o+})^{JK}$	$\frac{1}{2}\epsilon^{K PQ}(k_F)^{TJPQ} - \frac{1}{2}\epsilon^{JPQ}(k_F)^{TKPQ}$	3
$\tilde{\kappa}_{\text{tr}}$	$-\frac{2}{3}[(k_F)^{TXTX} + (k_F)^{TYTY} + (k_F)^{TZTZ}]$	1 <b>total: 19</b>
$k^1$	$(k_F)^{TYXZ}$	1
$k^2$	$(k_F)^{TXYZ}$	1
$k^3$	$(k_F)^{TYTY} - (k_F)^{XZXZ}$	1
$k^4$	$(k_F)^{TZTZ} - (k_F)^{XYXY}$	1
$k^5$	$(k_F)^{TXTY} + (k_F)^{XZYZ}$	1
$k^6$	$(k_F)^{TXTZ} - (k_F)^{XYYZ}$	1
$k^7$	$(k_F)^{TYTZ} + (k_F)^{XYXZ}$	1
$k^8$	$(k_F)^{TXXY} + (k_F)^{TZZY}$	1
$k^9$	$(k_F)^{TXXZ} - (k_F)^{TYYZ}$	1
$k^{10}$	$(k_F)^{TYXY} - (k_F)^{TZXX}$	1
$k_{(V)00}^{(3)}$	$-\sqrt{4\pi} (k_{AF})^T$	1
$k_{(V)10}^{(3)}$	$-\sqrt{4\pi/3} (k_{AF})^Z$	1
Re $k_{(V)11}^{(3)}$	$\sqrt{2\pi/3} (k_{AF})^X$	1
Im $k_{(V)11}^{(3)}$	$-\sqrt{2\pi/3} (k_{AF})^Y$	1 <b>total: 4</b>

TABLE XX: Lagrange density for the fermion sector of the minimal SME in Riemann-Cartan spacetime

Sector	Coeff.	Operator	Dim.	Factor	CPT	L.V.
Lepton		$\bar{L}_A \gamma^a \overleftrightarrow{D}_\mu L_A$	4	$\frac{1}{2} i e e^\mu_a$	+	
		$\bar{R}_A \gamma^a \overleftrightarrow{D}_\mu R_A$	4	$\frac{1}{2} i e e^\mu_a$	+	
	$(a_L)_{\mu AB}$	$\bar{L}_A \gamma^a L_B$	3	$-e e^\mu_a$	-	✓
	$(a_R)_{\mu AB}$	$\bar{R}_A \gamma^a R_B$	3	$-e e^\mu_a$	-	✓
	$(c_L)_{\mu\nu AB}$	$\bar{L}_A \gamma^a \overleftrightarrow{D}^\nu L_B$	4	$-\frac{1}{2} i e e^\mu_a$	+	✓
	$(c_R)_{\mu\nu AB}$	$\bar{R}_A \gamma^a \overleftrightarrow{D}^\nu R_B$	4	$-\frac{1}{2} i e e^\mu_a$	+	✓
Quark		$\bar{Q}_A \gamma^a \overleftrightarrow{D}_\mu Q_A$	4	$\frac{1}{2} i e e^\mu_a$	+	
		$\bar{U}_A \gamma^a \overleftrightarrow{D}_\mu U_A$	4	$\frac{1}{2} i e e^\mu_a$	+	
		$\bar{D}_A \gamma^a \overleftrightarrow{D}_\mu D_A$	4	$\frac{1}{2} i e e^\mu_a$	+	
	$(a_Q)_{\mu AB}$	$\bar{Q}_A \gamma^a Q_B$	3	$-e e^\mu_a$	-	✓
	$(a_U)_{\mu AB}$	$\bar{U}_A \gamma^a U_B$	3	$-e e^\mu_a$	-	✓
	$(a_D)_{\mu AB}$	$\bar{D}_A \gamma^a D_B$	3	$-e e^\mu_a$	-	✓
	$(c_Q)_{\mu\nu AB}$	$\bar{Q}_A \gamma^a \overleftrightarrow{D}^\nu Q_B$	4	$-\frac{1}{2} i e e^\mu_a$	+	✓
	$(c_U)_{\mu\nu AB}$	$\bar{U}_A \gamma^a \overleftrightarrow{D}^\nu U_B$	4	$-\frac{1}{2} i e e^\mu_a$	+	✓
	$(c_D)_{\mu\nu AB}$	$\bar{D}_A \gamma^a \overleftrightarrow{D}^\nu D_B$	4	$-\frac{1}{2} i e e^\mu_a$	+	✓
Yukawa	$(G_L)_{AB}$	$\bar{L}_A \phi R_B + \text{h.c.}$	4	$-e$	+	
	$(G_U)_{AB}$	$\bar{Q}_A \phi^c U_B + \text{h.c.}$	4	$-e$	+	
	$(G_D)_{AB}$	$\bar{Q}_A \phi D_B + \text{h.c.}$	4	$-e$	+	
	$(H_L)_{\mu\nu AB}$	$\bar{L}_A \phi \sigma^{ab} R_B + \text{h.c.}$	4	$-\frac{1}{2} e e^\mu_a e^\nu_b$	+	✓
	$(H_U)_{\mu\nu AB}$	$\bar{Q}_A \phi^c \sigma^{ab} U_B + \text{h.c.}$	4	$-\frac{1}{2} e e^\mu_a e^\nu_b$	+	✓
	$(H_D)_{\mu\nu AB}$	$\bar{Q}_A \phi \sigma^{ab} D_B + \text{h.c.}$	4	$-\frac{1}{2} e e^\mu_a e^\nu_b$	+	✓

TABLE XXI: Lagrange density for the boson sector of the minimal SME in Riemann-Cartan spacetime

Sector	Coeff.	Operator	Dim.	Factor	CPT	L.V.
Higgs	$\mu^2$	$\phi^\dagger \phi$	2	$e$	+	
	$\lambda$	$(\phi^\dagger \phi)^2$	4	$-\frac{1}{3!}e$	+	
		$(D_\mu \phi)^\dagger (D^\mu \phi)$	4	$-e$	+	
	$(k_\phi)^\mu$	$\phi^\dagger D_\mu \phi + \text{h.c.}$	3	$ie$	-	✓
	$(k_{\phi\phi})^{\mu\nu}$	$(D_\mu \phi)^\dagger (D_\nu \phi) + \text{h.c.}$	4	$\frac{1}{2}e$	+	✓
	$(k_{\phi W})^{\mu\nu}$	$\phi^\dagger W_{\mu\nu} \phi$	4	$-\frac{1}{2}e$	+	✓
	$(k_{\phi B})^{\mu\nu}$	$\phi^\dagger \phi B_{\mu\nu}$	4	$-\frac{1}{2}e$	+	✓
Gauge		$\text{Tr}(G_{\mu\nu} G^{\mu\nu})$	4	$-\frac{1}{2}e$	+	
		$\text{Tr}(W_{\mu\nu} W^{\mu\nu})$	4	$-\frac{1}{2}e$	+	
		$B_{\mu\nu} B^{\mu\nu}$	4	$-\frac{1}{4}e$	+	
	$(k_0)_\kappa$	$B^\kappa$	1	$e$	-	✓
	$(k_1)_\kappa$	$B_\lambda B_{\mu\nu}$	3	$e\epsilon^{\kappa\lambda\mu\nu}$	-	✓
	$(k_2)_\kappa$	$\text{Tr}(W_\lambda W_{\mu\nu} + \frac{2}{3}igW_\lambda W_\mu W_\nu)$	3	$e\epsilon^{\kappa\lambda\mu\nu}$	-	✓
	$(k_3)_\kappa$	$\text{Tr}(G_\lambda G_{\mu\nu} + \frac{2}{3}ig_3G_\lambda G_\mu G_\nu)$	3	$e\epsilon^{\kappa\lambda\mu\nu}$	-	✓
	$(k_G)_{\kappa\lambda\mu\nu}$	$\text{Tr}(G^{\kappa\lambda} G^{\mu\nu})$	4	$-\frac{1}{2}e$	+	✓
	$(k_W)_{\kappa\lambda\mu\nu}$	$\text{Tr}(W^{\kappa\lambda} W^{\mu\nu})$	4	$-\frac{1}{2}e$	+	✓
	$(k_B)_{\kappa\lambda\mu\nu}$	$B^{\kappa\lambda} B^{\mu\nu}$	4	$-\frac{1}{4}e$	+	✓
Gravity		$R$	2	$e/2\kappa$	+	
	$\Lambda$	1	0	$-e/\kappa$	+	
	$u$	$R$	2	$-e/2\kappa$	+	
	$(k_T)^{\lambda\mu\nu}$	$T_{\lambda\mu\nu}$	1	$e/2\kappa$	+	✓
	$s^{\mu\nu}$	$R_{\mu\nu}$	2	$e/2\kappa$	+	✓
	$t^{\kappa\lambda\mu\nu}$	$R_{\kappa\lambda\mu\nu}$	2	$e/2\kappa$	+	✓
	$(k_{TT})^{\alpha\beta\gamma\lambda\mu\nu}$	$T_{\alpha\beta\gamma} T_{\lambda\mu\nu}$	2	$e/2\kappa$	+	✓
	$(k_{DT})^{\kappa\lambda\mu\nu}$	$D_\kappa T_{\lambda\mu\nu}$	2	$e/2\kappa$	+	✓

TABLE XXII: Coefficients in the neutrino sector

Coeff.	Dim.	Oscillation	CPT	L.V.
$\tilde{m}_{AB}$	3	$\nu \leftrightarrow \nu, \bar{\nu} \leftrightarrow \bar{\nu}$	+	
$[(a_L)^\mu]_{AB}$	3	$\nu \leftrightarrow \nu, \bar{\nu} \leftrightarrow \bar{\nu}$	-	✓
$[H^{\mu\nu}]_{AB}$	3	$\nu \leftrightarrow \bar{\nu}$	+	✓
$[(c_L)^{\mu\nu}]_{AB}$	4	$\nu \leftrightarrow \nu, \bar{\nu} \leftrightarrow \bar{\nu}$	+	✓
$[g^{\mu\nu\sigma}]_{AB}$	4	$\nu \leftrightarrow \bar{\nu}$	-	✓

TABLE XXIII: Quadratic Lagrange density for the nonminimal photon sector in Minkowski spacetime

Coeff.	#	Operator	Dim.	Factor	CPT	L.V.
		$F_{\mu\nu}F^{\mu\nu}$	4	$-\frac{1}{4}$	+	
$(k_{AF}^{(3)})_\kappa \equiv (k_{AF})_\kappa$	4	$A_\lambda F_{\mu\nu}$	3	$\frac{1}{2}\epsilon^{\kappa\lambda\mu\nu}$	—	✓
$(k_{AF}^{(5)})_\kappa^{\alpha_1\alpha_2}$	36	$A_\lambda\partial_{\alpha_1}\partial_{\alpha_2}F_{\mu\nu}$	5	$\frac{1}{2}\epsilon^{\kappa\lambda\mu\nu}$	—	✓
$(k_{AF}^{(7)})_\kappa^{\alpha_1\alpha_2\alpha_3\alpha_4}$	120	$A_\lambda\partial_{\alpha_1}\partial_{\alpha_2}\partial_{\alpha_3}\partial_{\alpha_4}F_{\mu\nu}$	7	$\frac{1}{2}\epsilon^{\kappa\lambda\mu\nu}$	—	✓
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		
$(k_{AF}^{(d)})_\kappa^{\alpha_1\cdots\alpha_{(d-3)}}$	$\frac{1}{2}(d+1)(d-1)(d-2)$	$A_\lambda\partial_{\alpha_1}\cdots\partial_{\alpha_{(d-3)}}F_{\mu\nu}$	odd $d$	$\frac{1}{2}\epsilon^{\kappa\lambda\mu\nu}$	—	✓
$(k_F^{(4)})^{\kappa\lambda\mu\nu} \equiv (k_F)^{\kappa\lambda\mu\nu}$	19 + 1	$F_{\kappa\lambda}F_{\mu\nu}$	4	$-\frac{1}{4}$	+	✓
$(k_F^{(6)})^{\kappa\lambda\mu\nu\alpha_1\alpha_2}$	126	$F_{\kappa\lambda}\partial_{\alpha_1}\partial_{\alpha_2}F_{\mu\nu}$	6	$-\frac{1}{4}$	+	✓
$(k_F^{(8)})^{\kappa\lambda\mu\nu\alpha_1\alpha_2\alpha_3\alpha_4}$	360	$F_{\kappa\lambda}\partial_{\alpha_1}\partial_{\alpha_2}\partial_{\alpha_3}\partial_{\alpha_4}F_{\mu\nu}$	8	$-\frac{1}{4}$	+	✓
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		
$(k_F^{(d)})^{\kappa\lambda\mu\nu\alpha_1\cdots\alpha_{(d-4)}}$	$(d+1)d(d-3)$	$F_{\kappa\lambda}\partial_{\alpha_1}\cdots\partial_{\alpha_{(d-4)}}F_{\mu\nu}$	even $d$	$-\frac{1}{4}$	+	✓

TABLE XXIV: Spherical coefficients for the nonminimal photon sector in Minkowski spacetime

Type	Coeff.	Dim.	$n$	$j$	#
vacuum	$c_{(I)jm}^{(d)}$	even, $\geq 4$	—	$0, 1, \dots, d-2$	$(d-1)^2$
	$k_{(E)jm}^{(d)}$	even, $\geq 4$	—	$2, 3, \dots, d-2$	$(d-1)^2 - 4$
	$k_{(B)jm}^{(d)}$	even, $\geq 4$	—	$2, 3, \dots, d-2$	$(d-1)^2 - 4$
	$k_{(V)jm}^{(d)}$	odd, $\geq 3$	—	$0, 1, \dots, d-2$	$(d-1)^2$
vacuum orthogonal	$(\bar{c}_F^{(d)})_{njm}^{(0E)}$	even, $\geq 4$	$0, 1, \dots, d-4$	$n, n-2, n-4, \dots, \geq 0$	$\frac{(d-1)(d-2)(d-3)}{6}$
	$(\bar{k}_F^{(d)})_{njm}^{(0E)}$	even, $\geq 6$	$1, 2, \dots, d-4$	$n, n-2, n-4, \dots, \geq 0$	$\frac{(d-1)(d-2)(d-3)}{6} - 1$
	$(\bar{k}_F^{(d)})_{njm}^{(1E)}$	even, $\geq 6$	$1, 2, \dots, d-4$	$n+1, n-1, n-3, \dots, \geq 1$	$\frac{(d-4)(d^2+d+3)}{6}$
	$(\bar{k}_F^{(d)})_{njm}^{(2E)}$	even, $\geq 6$	$2, 3, \dots, d-4$	$n, n-2, n-4, \dots, \geq 2$	$\frac{(d-4)(d^2-2d-9)}{6}$
	$(\bar{k}_F^{(d)})_{njm}^{(1B)}$	even, $\geq 6$	$1, 2, \dots, d-4$	$n, n-2, n-4, \dots, \geq 1$	$\frac{d(d-2)(d-4)}{6}$
	$(\bar{k}_F^{(d)})_{njm}^{(2B)}$	even, $\geq 6$	$1, 2, \dots, d-4$	$n+1, n-1, n-3, \dots, \geq 2$	$\frac{(d+3)(d-2)(d-4)}{6}$
	$(\bar{k}_{AF}^{(d)})_{njm}^{(0B)}$	odd, $\geq 5$	$0, 1, \dots, d-4$	$n, n-2, n-4, \dots, \geq 0$	$\frac{(d-1)(d-2)(d-3)}{6}$
	$(\bar{k}_{AF}^{(d)})_{njm}^{(1B)}$	odd, $\geq 5$	$0, 1, \dots, d-4$	$n+1, n-1, n-3, \dots, \geq 1$	$\frac{(d+1)(d-1)(d-3)}{6}$
	$(\bar{k}_{AF}^{(d)})_{njm}^{(1E)}$	odd, $\geq 5$	$1, 2, \dots, d-3$	$n, n-2, n-4, \dots, \geq 1$	$\frac{(d+1)(d-1)(d-3)}{6}$